THE UNIVERSITY OF ALBERTA

THE RELATIONSHIP BETWEEN PRIMARY STUDENTS' RATIONALIZATION OF CONSERVATION AND THEIR MATHEMATICAL ABILITY AND ACHIEVEMENT

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "The Relationship Between Primary Students' Rationalization of Conservation and Their Mathematical Ability and Achievement" submitted by William George Cathcart in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

Piaget's concepts of identity, reversibility, and compensation were considered as possible ways of rationalizing conservation. The purposes of the present study were: (1) to investigate the frequency with which each mode of rationalization was used to justify conservation, (2) to investigate differences in some social and personal characteristics of subjects who exhibited different modes of rationalization, and (3) to examine the relationship between the different modes of rationalization for conservation and factors of mathematical achievement and mathematical ability.

One hundred twenty grade two and three students from twelve different schools were given a battery of tests including tests of conservation, mathematics achievement, mathematics ability, vocabulary, listening ability, and intelligence. Other variables included in the study were socio-economic status, age, grade, and sex. Out of the one hundred and twenty students in the sample, ninety-five were used in the analyses. The others were eliminated primarily because they were not able to conserve.

It was found that primary school children preferred to use identity-type arguments when they rationalized conservation. This was true of both total and partial conservers. However, partial conservers used compensation

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Many research projects concerned with Piagetian conservation have been undertaken in the past decade. From these studies enough evidence has accrued to indicate that the ability to conserve is related to general school achievement and to achievement in a number of curriculum areas.

Almy (1966) found that the ability to conserve was related to general school achievement. Pflederer (1964) found that the ability to conserve was related in a positive direction to children's performance in some musical tasks. Lansing (1966) found the same relationship in art work. An understanding of conservation was found by Rawson (1965) to be significantly related to reading comprehension. Studies by Reimer (1968), Pelletier (1966), Brace (1963) and others have shown that the acquisition of conservation contributes significantly to achievement in various aspects of the mathematics curriculum including an understanding of numerical concepts and measurement of length, area, volume, mass, and weight.

Piaget (1952) contends that "conservation is a necessary condition for all rational activity" (p. 3). In reference to mathematics in particular, Piaget (1952) asserts that:

> ... whether it be a matter of continuous or discontinuous qualities, of quantitative relations perceived in the sensible universe, or of sets and numbers conceived by thought, whether it be a matter of the child's earliest contacts with number or of the most refined axiomatizations of any intuitive system, in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding (pp. 3-4).

A typical task to test for the presence of conservation may run something like this: A child is confronted with two wooden rods of equal length and parallel to each other. The child is asked to confirm that the rods are indeed of equal length. When he is convinced of their equality, the experimenter moves one of the rods forward a few inches, maintaining their parallel status. The student is then asked to judge again the relative lengths of the two rods. Younger children frequently focus on the forward end of the moving rod and thus judge it to be longer after the transformation. Older children are able to consider several dimensions simultaneously and give the conservation response, "It is the same."

According to Piaget (1960), there are three thought processes which a child may use to arrive at the conservation answer, "It is the same." On the basis of Elkind's (1967) argument, these processes are perhaps better thought of as rationalizations of conservation. The first is illustrated by the child who says that it is

the same because "If you moved it back to where it was before it would be the same." This type of rationalization is called "reversibility" by Piaget. The second type of rationalization he terms "identity". A characteristic answer to illustrate this rationalization is, "You didn't add anything to it or take anything away, so it is the same." Another group of students may respond something like this: "This one is longer here but it is shorter here, so it is the same." Piaget calls this type of rationalization a "combination of compensated relations", or more simply, "compensation".

The present study was designed to explore further the trichotomous classification of verbal rationalizations for conservation in situations of mathematical significance.

I. STATEMENT OF THE PROBLEM

The purposes of the present study were: (1) to investigate the frequency with which each mode of rationalization (identity, reversibility, and compensation) was used to justify conservation, (2) to investigate differences in some social and personal characteristics of subjects who exhibited different modes of rationalization, and (3) to examine the relationship between the different modes of rationalization for conservation and factors of mathematical achievement and mathematical ability.

With respect to the first purpose it was proposed to investigate the possibility that a hierarchy of rationalization exists. This hierarchy could be based on simple frequency of use or it could be established on the basis of use by students at different levels of conservation. This hierarchy, if one exists, may have been established as a result of the type of experiences a child has had or it might be due to advancing cognitive structure. However, no attempt was made to explain why a hierarchy existed.

The social and personal characteristics referred to in the second purpose included intelligence, vocabulary, listening ability, conservation, grade, age, sex, and socio-economic status.

The third purpose stated above was the primary purpose of the present study. That is, it was intended to investigate the relationship between a student's mode of rationalization for conservation and his mathematical ability and his mathematical achievement in such areas as knowledge of geometric concepts, numeration, and the basic facts of arithmetic. It was anticipated that this would result in a hierarchy of the modes of rationalization in terms of how each contributed to achievement in various aspects of mathematics.

II. MAJOR HYPOTHESES

The three purposes stated above gave rise to the following null hypotheses.

Purpose One

Null Hypotheses

(1) There is no significant difference in the observed frequency with which each mode of rationalization is chosen and a rectangular frequency distribution expected by chance for:

(a) partial conservers,

(b) total conservers, and for

(c) all subjects.

(2) There is no significant relationship between the types of conservers (total or partial) and the mode of rationalization expressed for conservation.

(3) The mode of rationalization for conservation is independent of the degree of conservation. Degree of conservation was determined by the number of conservation responses made.

(4) The mode of rationalization is independent of the difficulty of the items on the conservation test. The item difficulty was determined by the proportion of students giving a conservation response to the item.

Purpose Two

Null Hypotheses

(5) There is no significant difference between subjects exhibiting different modes of rationalization for conservation on the following criteria:

(a) intelligence,

(b) socio-economic status,

(c) vocabulary,

(d) listening ability, and

(e) conservation.

(6) There is no significant relationship between mode of rationalization for conservation and:

(a) age,

(b) grade, and

(c) sex.

Purpose Three

Null Hypotheses

(7) There is no significant interaction between the mode of rationalization for conservation and intelligence, vocabulary, listening ability, socio-economic status, age, grade, and sex when mathematical ability and mathematics achievement are the criteria.

(8) There are no significant main effects due to mode of rationalization on the mathematics ability and

mathematics achievement tests.

(9) There are no significant differences among subjects who use different modes of rationalization for conservation in mathematical ability or mathematical achievement when intelligence, vocabulary, listening ability, conservation, socio-economic status, age, grade, and sex are controlled for separately.

(10) There are no significant differences among subjects who use only one mode, two different modes, three different modes, or all four modes of rationalization for conservation in mathematical achievement or mathematical ability.

III. DEFINITION OF TERMS

Invariant Transformation

An invariant transformation is the operation of changing the condition or configuration of an object without changing the specific physical properties being studied.

Conservation

A specific property of an object is said to be conserved if that property remains constant after an invariant transformation. For example, in the task described in the introduction, if a child says that moving the one rod forward does not change its length relative to the first rod he is said to conserve length.

Mode of Rationalization

The mode of rationalization is the verbal reason given by a subject to explain why a given property is conserved under an invariant transformation. As stated above, there are, according to Piaget, three such modes of rationalization; Reversibility, Identity, and Compensation. For the purposes of the current study, the identity mode has been divided into two modes. (See Chapter III for an explanation.) The two identity modes have been termed Substantive Identity and Operational Identity.

(1) Reversibility. Conservation is rationalized on the basis that a given transformation can be undone. That is, a transformation followed by its inverse would take us back to the starting point.

(2) Compensation. This term is used in the place of Piaget's phrase, "combination of compensated relations". In compensation, conservation is rationalized by combining two dimensions of the property after an invariant transformation. These dimensions compensate the one for the other, therefore, there is no change in the property.

(3) Substantive Identity. Conservation is rationalized on the basis that the invariant transformation has not changed the characteristics of the objects exhibiting the property.

(4) Operational Identity. Conservation is rationalized with the argument that an invariant transformation

has not added to or subtracted from the property in question.

Partial Conserver

A subject who scored from two through six on the conservation test was considered a partial conserver.

Total Conserver

A total conserver is a subject who was able to conserve on at least seven of the eight conservation tasks in the conservation test. This definition was formulated by considering the probability of getting seven or eight items correct just by guessing. The probability of this occurring by chance is about .05 using the binomial distribution.

Mathematics Achievement

A subject's achievement in mathematics was defined as the subject's score on the special achievement test designed for this study.

Mathematics Ability

A subject's mathematical ability was defined as his score on the Cooperative Primary Tests -- Mathematics, Form 12A.

Intelligence

A subject's intelligence was defined as his score

on the _Coloured Progressive Matrices_, Sets A, Ab, and B.

Vocabulary

A subject's vocabulary was defined as his score on the vocabulary section of the _Wechsler Intelligence Scale for Children_.

Listening Ability

A subject's listening ability was his score on the _Cooperative Primary Tests -- Listening_, Form 12A.

IV. DELIMITATIONS AND LIMITATIONS

The sample for the current study was randomly selected from a population of grade two and three children in ten schools in Edmonton, Alberta, and in two elementary schools in the County of Strathcona which is adjacent to the City of Edmonton.

The delimitations stated above impose some limitations on the generalizability of the results. It may be that older students exhibit a different mode of rationalization from the subjects used in this study. Rural students may also rationalize conservation differently from the urban students in the current sample because of different academic and social experiences. These limitations on the generalizability of the results were not controlled for. It will be left to further research to determine whether or not the differences mentioned above do exist.

The use of one classification scheme for the modes of rationalization for conservation may also limit the present study because responses may fit better into a different classification system than that suggested by Piaget.

Another limitation of the present study is that it was static. Therefore, no change in the mode of rationalization of conservation with time was indicated.

The study reported here was further limited in that no effort was made to control for the effects of previous learning which may have affected how a subject rationalized conservation.

V. ASSUMPTIONS

One major assumption of the present study was that each mode of rationalization could be applied logically to each task on the conservation test. It may be that all four modes of rationalization are not equally feasible for all age levels on all tasks. However, each mode of rationalization was possible.

Another assumption was that the mode of rationalization expressed was the rationalization believed. This is an assumption that is unavoidable with all forms of verbal testing.

It was also assumed that no learning took place from one item on the conservation test to the next. Students

were not told whether they were correct or incorrect and, furthermore, the test took only about fifteen minutes to administer. Therefore, any learning that might have taken place would be minimal. However, if learning did take place, no serious limitation was imposed as the mode of rationalization expressed was the main concern, not the actual score on the test.

VI. SIGNIFICANCE OF THE STUDY

It was shown in the introduction that conservation is an important factor in the understanding of many of the mathematical concepts taught in the primary school. Therefore, it is logical that conservation should be attained before the mathematical concepts are taught. However, not all students are able to conserve even number by the time they reach grade one. Therefore, the teaching of conservation becomes an important consideration. If students can be taught to conserve, then concepts depending upon conservation may be taught at an earlier age if such is considered desirable. This raises the question, "What is the best method of teaching conservation?" Fortunately, this is the direction much of the research based on Piaget's theory has taken in recent years, but, unfortunately, there has been only limited success. However, a recent study by Towler (1967) has implications for the significance of the current study.

Towler's main hypothesis was that learning of conservation could take place if the subjects were presented with experiences which would make meaningful the crucial aspects of conservation. These crucial variables correspond to the modes of rationalization discussed earlier. Towler worked with continuous quantities and his training sessions were designed to provide three types of understandings: (1) an understanding that a fluid retains its identity during a transformation, (2) an understanding of the compensatory relationships between height and width, and (3) an understanding of the principle of reversibility.

While Towler postulated that identity, reversibility, and compensation are crucial factors in the acquisition of conservation, he made no effort to see if each is as important as the others. He does not say so, but it is implied in his training session that he considers reversibility to be of little importance since he teaches directly the concepts of identity and compensation but does not train for reversibility at all. This is striking in that Piaget (1960) seems to consider reversibility as the most important factor in conservation when, in discussing the three types of responses, he says, "and this especially" (p. 140) when referring to reversibility. The current study should help resolve the role of reversibility by establishing a hierarchy of response types if such a

hierarchy exists. However, the main significance lies in establishing a hierarchy of rationalizations on the basis of their importance to various mathematical learnings. In other words, the hierarchy of importance with respect to mathematical achievement may be different for numeration than for geometry. Thus, the outcome of the present study may be that each mode of rationalization is of major importance for some area of the mathematics curriculum. If such is the case the results should be significant for designers of primary mathematics curriculum.

In brief, since conservation is important for learning in general and for mathematics learning in particular, it is important that children be given opportunity to improve their ability to conserve in areas important in mathematics such as number, length, area, and volume. The experiences provided students in the primary grades are important in this respect. But different students possess different cognitive structures and learn best by different approaches. Therefore, in devising experiences for children in mathematics it is important that the characteristics of students who rationalize conservation differently be known so that we can predict better what type of experiences to provide for an individual student to speed up his acquisition of conservation.

There are many studies that show that the ability to conserve is related to achievement. (See Chapter II.)

However, these studies do not generally relate the ability to conserve to various dimensions of achievement. It may be that the presence of conservation is related to achievement in only some areas of mathematics or it may be that conservation of particular properties is related to achievement in a related area. For example, the ability to conserve number may have no relationship to a student's achievement on a test of linear or cubic measure but it may be highly related to his achievement on a test of the principles of our numeration system. The present study investigated the relationship between conservation in general and various factors of mathematics achievement but it went beyond this and also investigated how a student's rationalization for conservation was related to various achievement factors. A knowledge of these relationships should help teachers to diagnose student difficulties in mathematics more accurately and enable them to provide more effective remedial activities.

Similarly many studies have found that intellectual ability is closely related to the ability to conserve. While the current study examined this relationship, the main concern was more with mathematical ability than with intelligence. The relationship between mathematical ability as measured by the _Cooperative Primary Tests -- Mathematics_ and a student's ability to conserve and in

particular his rationalization for conservation were investigated. The above relationship is primarily of theoretical importance but it is also important for the classroom teacher and school psychologist. For example, if it was found that students who rationalized conservation with compensatory arguments scored high on the geometry dimension, then an individual child's ability in the geometry area may be improved if he was given instruction in compensatory relationships.

Further justification for the current research lies in the fact that little is currently known about the ways in which the transition from nonconservation to conservation takes place (Wohlwill and Lowe, 1962). While this study does not supply this lacking information, it may serve as one of the steps in that direction if partial conservers use different modes of rationalization than total conservers. Wohlwill and Lowe suggest that special attention should be paid to the types of justifications children give at various stages of chronological development.

CHAPTER II

REVIEW OF RELATED LITERATURE

One of the purposes of this chapter is to look in a brief and general way at Piaget's theory of cognitive development and more specifically, to show how the concepts of identity, reversibility, and compensation arise from Piaget's theory.

After establishing the origin of the concepts of identity, reversibility, and compensation, some literature will be discussed which indicates how the above concepts influence the acquisition of conservation which in turn, according to some of the research, affects achievement in various aspects of mathematics.

Figure 1 shows in schematic form the outline of Chapter II. The current study fits into this outline by attempting to short circuit (dotted arrow) the usual flow of ideas (solid arrows). That is, while much research has been concerned with the relationship between conservation and mathematics achievement, the present study purports to show a relationship between the identity, reversibility, and compensation modes of rationalization and mathematics achievement.

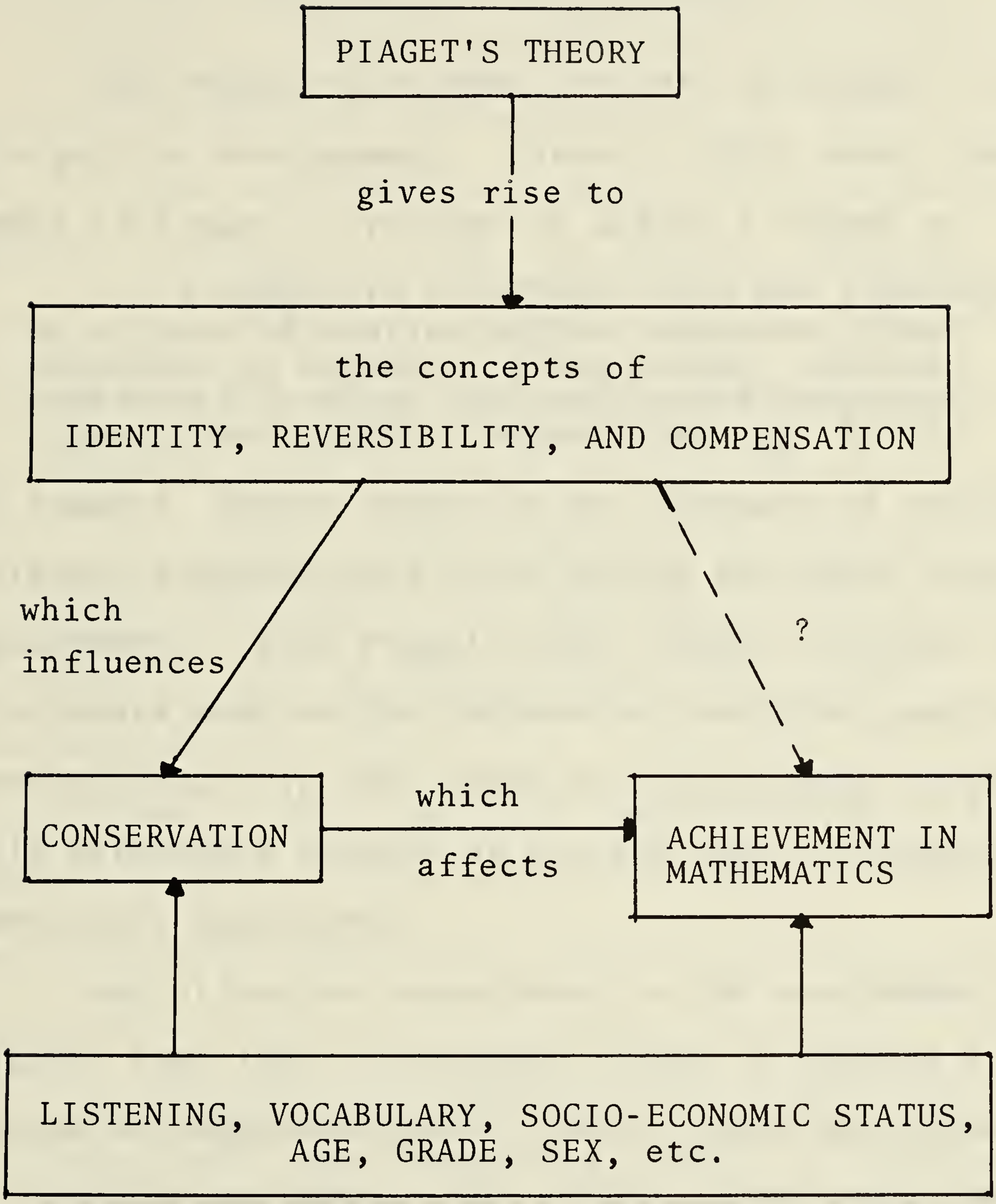


FIGURE 1

OUTLINE OF CHAPTER II IN SCHEMATIC FORM

I. JEAN PIAGET'S THEORY

The concept of schema is basic to Piaget's theory of cognitive development. Flavell (1963) draws from a number of Piaget's writings to define a schema as:

> ... a cognitive structure which has reference to a class of similar action sequences, these sequences of necessity being strong, bounded totalities in which the constituent behavioral elements are tightly interrelated (pp. 52-53).

For example, Piaget speaks of the schemata of sucking, striking, grasping, and so on during the early stages of development. Later Piaget (1952) refers to other types of schemata such as the "schema of intuitive qualitative correspondence" (p. 88) which is the strategy by which a child determines whether or not two sets of objects are numerically equivalent.

Any situation encountered in the environment which requires some type of response action is handled by a process of adaptation which Piaget (1960) describes as an "equilibrium between the actions of the organism on the environment and vice versa" (p. 7).

Piaget says that adaptation takes two forms, assimilation and accommodation. Harrison (1967) succinctly states how these two processes operate.

> Any problematic situation requiring behavior which is already generally represented in cognitive structure by a schema is handled by being assimilated to the schema. If the individual has no completely relevant schema, new behavior sequences are built by experi-

> mentation or instruction, or both, to enable existing schemata to accommodate to the new situation. Adaptation of an individual to his environment results from the interplay of assimilation and accommodation (p. 47).

As the child develops, assimilation and accommodation come more and more to be in equilibrium. However, Piaget's theory of equilibrium is more than simple balance between assimilation and accommodation. Lefrançois (1967) points out that at a more complex level equilibrium is:

> ... a state of development with respect to mental structures where external disturbances can be compensated for within the system of structures without either destroying the essential logical properties of that system or distorting the reality which is being reacted to (p. 165).

Piaget (1964) feels that equilibration is the most fundamental factor in development. (Even more fundamental than maturation, experience, or social transmission.) He says that:

> ... in the act of knowing, the subject is active, and consequently, faced with an external disturbance, he will react in order to compensate and consequently he will tend towards equilibrium. Equilibrium, defined by active compensation, leads to reversibility. Operational reversibility is a model of an equilibrated system where a transformation in one direction is compensated by a transformation in the other direction. Equilibration ... is thus an active process. It's a process of self-regulation (pp. 13-14).

Schema, assimilation, accommodation, and equilibrium are key concepts in Piaget's theory of intellectual development, which development he claims takes place in four major stages.

Sensori-Motor Stage (Birth to 2 or 3 years of age)

The only schemata in the intellectual structure of a newborn infant are reflex actions. These actions are discrete and unrelated. Gradually, after experiences with objects these schemata become coordinated into a sequential set of actions with respect to a given object. However, objects as yet do not have a permanence since the object exists for the child only if he perceives it. It is only when the child has internalized the action or schemata associated with an object that he is able to think of the object without acting on it or perceiving it. This internalization of action is the basis for symbolic thought and, consequently, of language. Therefore, this internalization of action is the most significant intellectual development of the sensori-motor stage. The child's thinking in this stage of development is very egocentric but the foundation has been laid for further development via assimilation and accommodation to his immediate environment.

Pre-Operational Stage (From 2 or 3 to 7 or 8 years of age)

Piaget divides this stage into two subperiods; the first of which he calls pre-conceptual thinking. This period ranges from about two to four years of age. The second subperiod is that of intuitive thinking which covers the years from about four to seven or eight.

In the pre-conceptual substage, memory becomes possible and the child can now think of things in their absence. However, he lacks a true concept of class. He can identify members of a class but cannot group objects into their classes, thus the term pre-concept. For example, on a walk through the woods the child does not know whether he observes a succession of different snails or whether the same snail keeps reappearing. All snails are the same to him.

This first substage of the pre-operational period of development is sometimes called the transductive period rather than the pre-conceptual period because the child's thinking is transductive, that is, from particular to particular, rather than inductive or deductive.

The second substage of pre-conceptual thought, the intuitive substage, is the first stage in which we are particularly interested since some of the subjects used in this study were still in this substage. Piaget's conservation experiments show clearly that during the intuitive substage perception is still a powerful force in the child's thinking which prevents him from applying rules of logic in a problem situation. For example, Piaget presented the child with some red and green beads of equal size and asked him to drop one of each color into two equal-sized glass containers simultaneously. After about six beads were dropped into the containers, Piaget

asked the child if there were the same number of beads in both containers. If the child agreed, then the beads from one of the containers were poured into another container which was taller and thinner. The child was asked again if there were the same number of beads in both containers. In the early part of the intuitive sub-stage perception was so strong that most subjects could not conserve the idea of six. Some would say there were more beads in the second container because it was higher while others would say there were fewer because it was thinner. Perception does not allow the child to logically consider both dimensions simultaneously. Perception prevents conservation even though the subjects were able to count the beads. A slightly more advanced child begins to have doubts as evidenced by statements like, "There must be more because it's higher, but it's thinner also." After this period of vacillation, the child eventually realizes that height and cross section must be considered simultaneously. When this stage is reached he is able to conserve and to think operationally which is the next period of development.

<u>Concrete Operational Stage</u> (from 7 or 8 to 11 or 12 years)

In the concrete operational stage of development the child's thinking loses much of its ego-centric qualities and is not as susceptible to perceptual cues as it was in

the earlier stages. Action becomes more internalized which results in the ability to classify, serialize, and work with numbers. This development is due to the formation of "groupings". Piaget (1960) describes a "grouping" as a "certain form of equilibrium of operations, i.e., of actions which are internalized and organized in complex structures ..." (p. 36).

Piaget goes on to list five conditions for "groupings" or equilibrium of internalized action to be developed. In other words, he lists five rules of logic used by the child in the concrete operational stage. These are:

(1) Combinativity. "Any two elements of a grouping may be combined and thus produce a new element of the same grouping" (Piaget, 1960, p. 40). This property is better known as closure. The child knows that adding two numbers yields a third number.

(2) Reversibility. For any operation there is an inverse operation which cancels the first.

(3) Associativity. It makes no difference in what order three or more operations are combined.

(4) Identity. The awareness that there are operations which leave objects unchanged.

(5) Tautology or iteration. (a) Tautology. The repetition of a class or a relation does not change it. (b) Iteration. The repetition of a number yields a different number.

When a child applies these rules to conservation problems he will be able to conserve the property in question. Piaget (1960) asserts this very strongly.

> ... assuming that the intuitive relations ... are at a certain moment suddenly "grouped", the first question is to decide by what internal or mental criterion grouping is to be recognized. The answer is obvious: where there is "grouping" there will be the conservation of a whole, and this conservation itself will not merely be assumed by the subject by virtue of a probable induction, but affirmed by him as a certainty in his thought (p. 140).

Piaget immediately goes on to say that if we ask the child his reasons for conservation the child will respond in one or more of three ways. He might say that "nothing has been removed or added" (Ibid.), "Or else he replies that the height makes up for the width lost ..." (Ibid.), "Or else, and this especially, he replies that a transfer from A to B may be corrected by a transfer from B to A ..." (Ibid., p. 140-141). These three responses will be recognized as identity, compensation, and reversibility respectively according to the definitions given in Chapter I. The above discussion by Piaget provides justification for the categories of rationalization used in the current study. A description of the categories used is given in Chapter III.

The child is unable, however, to apply the "grouping" principles to all types of conservation problems at the same time. Conservation of mass or quantity usually is

attained around the ages of seven or eight, conservation of weight around the ages of nine or ten, and conservation of volume only at the end of the concrete operational stage or around the ages of eleven or twelve (Flavell, 1963).

In summary, the child, during the concrete operations stage of development, learns to deal with concepts of class, relation, and number. The ability to conserve is the key criterion used to determine the presence of a child in this stage of development. However, the principles of conservation emerge later for some properties than for others. The child's thinking still lacks complete generality because he is able to deal with only real and visible objects or at least objects capable of evocation. Thus his reasoning is tied to his own concrete experience.

Formal Operations (from 11 or 12 years of age and over)

During the formal operations stage the ties that restricted the child's reasoning to the real and concrete are broken and he is able to deal with the hypothetical. He is able to consider hypotheses which may or may not be true and deduce what would happen if they were true. Lefrançois (1967) draws upon a number of the main concepts of Piaget's theory mentioned at the outset of this chapter to summarize the development of cognitive structures during the formal operations period.

> This period is the culmination of an adaptive process which began at birth, and which, through the processes of assimilation and accommodation, has resulted in the development of intellectual structures which are theoretically as sufficient for understanding and coping with the environment as they ever will be (p. 170).

II. ROLE OF IDENTITY, REVERSIBILITY, AND COMPENSATION IN CONSERVATION

In the preceding section it was shown that Piaget considers identity, reversibility, and compensation to be essential mental capabilities (or at least rationalizations) for the acquisition of conservation. Most studies that have considered the role of identity, reversibility, and compensation in conservation have been studies designed to induce conservation by various training techniques at an earlier age than it would normally appear.

One of the first attempts to induce conservation was conducted by Smedslund (1961, a,b). He termed his approach "cognitive conflict". In his approach Smedslund used two identical pieces of plasticine, one piece was transformed in some way while from the other piece of plasticine a small quantity was either added or subtracted and placed in the child's view. Cognitive conflict was assumed to be induced if subtraction from one piece of plasticine was observed and the child also thought that the transformation decreased the quantity or weight of the other piece of plasticine. Conflict was also induced when

the addition of a quantity of plasticine to one of the pieces was accompanied with a perceptual increase in the other piece of plasticine due to the transformation. Referring to these situations Smedslund (1961a) says:

> The child has to reach a decision as to the relative size of the two changes, and the state of inner conflict and uncertainty preceding this decision may well have the effect of inducing pronounced cognitive changes, especially if the same type of situation is repeated (p. 157).

While Smedslund's results are not very convincing in terms of success at inducing conservation in non-conservers, the responses of some of his subjects who had acquired conservation as a result of the training indicate that identity seems to play a role in conservation. Smedslund quotes the rationalizations used by four of his subjects to justify their conservation response on the post-test. The rationalizations used were:

(1) ". . . because you haven't taken any plasticine away."

(2) "Because you have not added anything."

(3) "Because if you take two equal (objects) they have to be equal all the time."

(4) "Because you did not take anything off that one." (Smedslund, 1961a, p. 159).

The first, second, and fourth responses fit the operational identity category defined in Chapter III of the present study while the third response is representative of the substantive identity category defined for the current study.

Wohlwill and Lowe (1962) used four different training techniques. One of their training groups was a control and one of the other groups was trained by what Wohlwill and Lowe termed the addition and subtraction method. Their expectation was that the latter method would prove to be superior to the others in inducing conservation. The addition and subtraction group received training which was analogous to Smedslund's addition/ subtraction training except that Wohlwill and Lowe were concerned with conservation of number instead of substance and weight. This technique provided training in identity because the child saw the loss of identity when something was added or subtracted. It was assumed in the training method that the child would infer that identity was maintained when no addition or subtraction had occurred.

While Wohlwill and Lowe found no significant differences among their four training groups with respect to the learning of conservation, they did find that the greatest gains were made by subjects in the addition and subtraction group. This lends some support to the position that an awareness of identity is an important factor in the acquisition of conservation.

Nair (cited from Bruner, *et al.*, 1966) used forty five-year-old kindergarten subjects in an experiment designed to discover if a judgment of identity affected how subjects viewed the equivalence of two continuous quantities.

She also devised a training procedure in which the identity of a quantity of water moved to different containers was labelled. Children were questioned as to the equality of the amount of water after it was poured into a different sized container and also as to the sameness of the water (i.e., identity). Nair found that subjects who conserved quantity also believed that it was the same water but the converse was not necessarily true. On the basis of her findings Nair tentatively concluded that "a recognition of identity is a necessary if not a sufficient condition for the recognition of quantitative equivalence" (p. 189).

Frank (cited from Bruner, et al., 1966) also worked with conservation of continuous quantities. He found that placing a screen between the subject and the different sized beakers into which the water was poured helped children to conserve quantity. Frank claimed that the screen "forces" the child to make a judgment based on an identity argument since perceptual cues do not influence the child.

Wallach and Sprott (1964) provided training in a reversibility context. They hypothesized that a child will have acquired conservation of number when he realizes that an inverse operation will cause a particular property to reappear at the same value it had before the transformation. They asked their first grade subjects to predict whether or not a one-to-one correspondence could be re-established

between a group of six dolls and six beds after an additive, subtractive, or spatial transformation had been carried out. A second part of the experiment used five checkers matched with five three-inch by five-inch cards. Subjects were asked to reconstruct the one-to-one correspondence in both instances. The researchers found that the experimental group of fifteen nonconservers trained in reversibility significantly out performed a control group on a conservation of number test. The superiority of the trained group over the control group was still highly significant (p = .00002) on a delayed post-test two to three weeks after the experiment.

Wallach and Sprott argue that their success cannot be explained in terms of direct observation (counting) or social reinforcement. It is of interest with respect to the present study that Wallach and Sprott also argue that a recognition of compensatory relationships or that an "inference from change with addition and subtraction" (p. 1067) cannot account for their results. The latter is a direct reference to Smedslund's work. Wallach and Sprott concluded that:

> ... the recognition of reversibility ... is the only interpretation of the development of conservation in the current literature which provides a tenable explanation of the results of this experiment (p. 1067).

This conclusion seems to agree with Piaget (1960) who admits that while "younger children have already on

occasion admitted the possibility of a return to the starting point", reversibility is nevertheless the key to the acquisition of conservation (p. 141).

Wallach, Wall, and Anderson (1967) extended the earlier study by Wallach and Sprott. They used the same type of subjects, namely first grade students from a middle class school. One of the training procedures used by Wallach, Wall, and Anderson provided experience with reversibility including the dolls and beds task used in the earlier study. In addition they trained another group of subjects by giving them experiences with addition and subtraction.

The results of Wallach, Wall, and Anderson's study again indicated the ability of the reversibility training procedure to induce conservation of number. Twelve of the fourteen subjects in the reversibility training session changed from nonconservation to conservation of number on an immediate post-test. All twelve also gave conservation responses on a delayed post-test, however, two of these subjects succumbed on the delayed post-test to the experimenter's suggestion against conservation.

The addition-subtraction training procedure was not successful at inducing conservation. The role of reversibility in the acquisition of conservation is still defended by Wallach, Wall and Anderson as a result of their study. However, its importance is somewhat reduced by the finding

that very few of their subjects referred to reversibility when they were asked to justify conservation responses. The authors suggest that perhaps the doll reversibility-training procedure may have been effective because it helped the subjects to stop relying on misleading cues. However, they still argue for the role of reversibility in conservation and finally conclude that "in order for a child to conserve, he must both recognize reversibility and not rely on inappropriate cues" (p. 441).

Pufall (1967) investigated the effects of three different training procedures on the conservation of spatial order in five and six-year-old children. Pufall describes his training groups as follows:

> Reversibility training demonstrated concretely the operation of reversibility while involving the child actively in the manipulation of order. Reproduction training emphasized only manipulation of order. Concept Identification, a sorting task, minimized motoric involvement but emphasized discrimination of instances of "reversed" from other "different" ordering (p. 4583).

Apparently all three groups learned rapidly but the reversibility group out performed the other two groups on a four-task post-test. The results confirmed Pufall's prediction that the reversibility training would be the best of the three methods for inducing conservation of spatial order. He concluded that:

> The superiority of the reversibility group suggested that it is concrete experience with reversibility and not merely manipulation with

> or improved discrimination of reversed order which "normally" leads to operational understanding of spatial order (p. 4584).

Beilin (1965) and Smith (1968) both used a training procedure which involved the giving of a verbal rule for conservation along with a demonstration of reversibility. Both studies found this technique to be highly successful. Beilin worked with conservation of length and number while Smith was concerned with the conservation of weight. The success of their method lends some additional support to the importance of reversibility in the acquisition of conservation. Beilin's and Smith's studies in themselves are not an adequate basis on which to make this conclusion because of the verbal rule factor but, in conjunction with the other studies reported, they help confirm that the recognition of the reverse transformation is important in conservation.

While no studies seem to focus solely on the role of compensation in conservation a number of studies do treat it in conjunction with identity and/or reversibility. Bruner (1966) seems to see the necessity of considering the role of compensation when upon discussing Nair's study suggests that "general intellectual growth may depend to some extent on sheer 'channel capacity', the ability to register on several aspects of the situation at once" (p. 192).

Another experiment reported by Bruner (1966) conducted by Carey examined further the role of reversibility and

compensation in conservation. Nonconserving four and five year old children were confronted with two identical beakers, one partly filled, the other empty. Then the subject was shown a set of five pairs of partially filled beakers and asked to select the one with the same amount of water as there was in the original beaker. (None of the new beakers was the same size as the original pair.) This experiment was unsuccessful in improving conservation of quantity. However, Carey found that about one-third of her subjects resorted to some form of compensatory argument to defend their selection. This would suggest that even pre-school children can use a compensatory rationalization, that is, they can focus on more than one dimension simultaneously.

Sigel, Roeper, and Hooper (1966) trained a small sample of four to five year old subjects in what they called the prerequisites of conservation, multiple classification, multiple relationality, and reversibility. Through a structured teaching session involving verbalization and demonstration, Sigel, Roeper, and Hooper attempted to bring to the attention of their subjects the fact that:

> ... objects have multiple characteristics (multiple classification), that these can be combined in various ways to produce new categories (multiple relations), and that categories of objects can be recognized and brought back to the original (reversibility) (p. 300).

Multiple classification was designed to help the children label objects in different ways, that is, to establish the "identity" of the object. In multiple relations the subjects had to consider several dimensions of a classification scheme and therefore this aspect of their training could be likened to compensation.

The results of the above study led to the conclusion that the training session resulted in significant improvement in the conservation of substance and weight. This lends some additional support for the role of identity, reversibility, and compensation in the acquisition of conservation.

Towler (1967) hypothesized that identity, reversibility, and compensation are the keys that will unlock the door to conservation. Consequently he devised a training session in which mainly identity and compensatory relations were presented to nonconserving and partially conserving grade one subjects.

To provide training in the identity relationship, Towler asked two types of questions. A drink was poured from one container into a metal can and the subjects were asked, "Does pouring change the kind of drink?" The second type of question asked, "Does pouring change how much (amount) there is to drink?"

A second part of the training session was designed to confront the subjects with the compensatory relations of

height and width. Containers of various dimensions were used so that the levels of the liquids varied. Subjects were asked to predict where the levels would be and to explain why the level was higher or lower in a given container than in the original standard.

The final aspect of the training session was concerned with the ability of the child to relate what he had just discussed to pictorial representations of similar situations.

Towler claimed to be highly successful in inducing conservation of continuous quantity in his experimental group which helps substantiate the importance of the awareness of identity and compensation in the acquisition of conservation.

This section has outlined some research which seems to point to the important role played by the concepts of identity, reversibility, and compensation in the acquisition of conservation. With respect to these concepts, one question is important for the present study. Although identity, reversibility, and compensation are important concepts in conservation are children able to verbalize these concepts? Some of the research referred to in this section helps answer this question in the affirmative.

The work of Smedslund (1961a) was referred to earlier and a sample of four of the responses made by some of his subjects indicated that they could verbalize the

concept of identity.

Wallach and Sprott (1964) were able to place justifications for conservation into one of four categories.

(1) The objects fit together or were the same number before.

(2) The objects could be fit back together again.

(3) Nothing was added or taken away; the objects were only rearranged.

(4) One set was closer together than the other set, or an object in one set was between two in the other.

The first category used by Wallach and Sprott is what has been termed substantive identity in the present study. The second category listed above parallels the reversibility category as defined in Chapter III. The last two categories used by Wallach and Sprott conform to the operational identity and the compensation categories respectively of the present study.

Bruner (1966) reports that in Nair's study fifty per cent of the subjects used some form of "It's the same water" as their main argument for conservation of amount. This is a verbalization of the concept of identity. Similarly in Carey's study of the conservation of liquids, Bruner observed that the two most frequent justifications for equality were, "It's only the same water", and "You only poured it", which are representative of the substantive

identity and operational identity categories respectively. However, some other responses in the same study were placed in a category termed "conflict". A sample response from this category was, "It goes down because it is wider". According to the category descriptions given in Chapter III of the present study the latter response would fit the compensation category.

Nine categories were used by Wallach, Wall, and Anderson (1967) to classify justifications for conservation of number or amount. Their first category, "Equality Before" referred to the fact that every bed had a doll in it before the transformation was carried out. This fits the definition of the substantive identity category used in the present study. The operational identity category used in the current study is similar to the one termed "Addition-Subtraction" by Wallach, Wall, and Anderson. Responses in this category referred to the fact that nothing had been added or taken from the property. Wallach, Wall, and Anderson's "Reversibility", and also their "Matching" categories are included in the reversibility category of the present study. Their matching category was like the reversibility one except that the inverse transformation was not actually carried out. That is, the child just verbally matched dolls and beds by saying something like, "this goes with this, this goes with this", etc.

A category termed "Closer or Wider" by Wallach, Wall, and Anderson conforms to the compensation category used in the present study in that several compensating dimensions of a configuration are referred to.

The remaining four categories used by Wallach, Wall, and Anderson conform to the other rational responses, irrational responses, and no response categories used in the present study. These latter categories are not of any immediate interest.

Before leaving this section it is interesting to note that the studies referred to used a variety of conservation tests. Towler (1967) and the three studies cited from Bruner (1966) used conservation of continuous quantity as their criterion. Wohlwill and Lowe (1962), Wallach and Sprott (1964), and Wallach, Wall, and Anderson (1967) used conservation of number tests. The latter study also used continuous quantity. Pufall (1967) used conservation of spatial order as the criterion while Beilin (1965) used conservation of length and Smith (1968) used conservation of weight as the criterion. Siegel, Roeper, and Hooper (1966) and Smedslund (1961a) used the conservation of substance and weight as their criterion.

Although each of the above studies was concerned with a specific type of conservation when it is considered as a whole, the implication is that an awareness of identity, reversibility, and compensation are important

in the acquisition of all types of conservation. Therefore, a test of conservation for which rationalizations of conservation are asked for could be a general test involving several different types of conservation. All three types of rationalizations, identity, reversibility, and compensation seem to be given for all types of conservation problems.

III. THE ROLE OF CONSERVATION IN MATHEMATICS ACHIEVEMENT

In this section a number of studies will be briefly reviewed which indicate that the ability to conserve has some bearing on achievement in mathematics.

Dodwell (1961) conducted a study in Kingston, Ontario in which he examined children's understanding of number concepts. His test of number concepts included correspondence, seriation, cardination, and ordination as well as conservation. However, all the items were based on Piaget's work. Dodwell correlated the scores on this test with scores on a teacher-made test of achievement in mathematics administered to the same thirty-four kindergarten students. The latter test was administered approximately six months after the number concepts test. Dodwell found a correlation of 0.59 between the two tests which seems to indicate some association between conservation and achievement. However, the time lapse between the two tests and the possible poor reliability

and/or validity of the teacher-made test limits the value of Dodwell's results.

Hood (1962) also devised a test consisting of Piagetian-like items. His test included conservation of number and quantity, seriation, correspondence, and additive composition of classes and numbers. Hood compared the performance of one hundred and twenty-six four to six year old subjects on the above test with teachers' ratings of the mathematical ability of the subjects. A significant and positive relationship was observed. However, this result is also suspect since teachers' ratings of achievement in arithmetic skills is not a very reliable or valid instrument.

Both a cross-sectional and a longitudinal study of concept development in middle and lower class subjects in the New York area was conducted by Almy (1966). One of the purposes of the study was to investigate the relationship between conservation of number and continuous quantity and achievement in a number of curriculum areas including mathematics. Achievement in mathematics was determined by scores on the <u>New York Inventory of Mathematical Concepts</u>.

Almy's report of her cross-sectional study does not clearly indicate the results but she claims that:

> ... children who perform well in the conservation tasks also do well in beginning reading and arithmetic. Such an advantage is greater in kindergarten and first grade than it is later when conservation ability becomes more widespread (p. 71).

The data that Almy does report seem to indicate that the above relationship is more pronounced in the middle class children than in the lower class group.

In the longitudinal part of her study Almy interviewed the same subjects (except for those who had moved away or left school for other reasons) five times from kindergarten through grade two. She correlated mathematics achievement at the end of her study with the "sum of standard scores representing each child's performance in relation to that of his peers in his particular school at five successive interviews" (p. 101). The standard scores were on the conservation test.

Using the above procedure Almy observed correlations from a low of 0.26 on the premeasurement section of the mathematics test to a high of 0.53 on the Numerical Concepts section. These correlations were obtained for the middle class subjects. The respective correlation coefficients for the lower class children were 0.41 and 0.38. Almy felt that these correlation coefficients were high enough to indicate that there was some relationship between the ability to conserve and mathematics achievement. However, she adds that further study of this relationship is needed.

Steffe (1966) devised a conservation of number test which was different from the usual Piaget-type conservation test in that no one-to-one correspondence was used to establish initial equality between a standard row of objects

and a second row and no transformation was applied to one of the rows. Steffe merely set out two sets of objects arranged so as to appear either equal or unequal in number. The children could use whatever techniques they wished to arrive at an answer with respect to the equality of the two sets. Steffe had twelve tasks of this nature.

The twelve-item conservation test was used to group first grade subjects into four conservation categories. Intelligence was controlled by also placing each subject into one of three intelligence categories. In the analysis, Steffe compared the mathematical achievement of the subjects in the four conservation categories within each intelligence level. The analysis indicated that children who had the lowest conservation scores had significantly lower scores on a test of addition problems and addition basic facts than subjects in the other three levels of conservation.

Reimer (1968) used a test of conservation of number, quantity, and length and the Seeing Through Arithmetic Test 1 to correlate conservation and mathematics achievement. Reimer found that there was a significant correlation ($r = .41$) between conservation scores and scores on the mathematics test. One interesting finding in Reimer's study was that a high mathematics achievement score could be predicted for subjects who had high scores on the conservation test but no prediction was possible for non-conservers or at least for subjects who had low scores on

the conservation test. Some of these poor conservers also scored high on the mathematics achievement test while some scored poorly.

One study which examined the relationship between mathematics achievement and conservation of substance found no statistical relationship. This study was conducted by Overholt (1964). In all his analyses, Overholt controlled for intelligence. This raises another issue which will be pursued in the next section of this chapter. In the meantime the literature seems to indicate that the ability to conserve helps children to achieve in mathematics. This is a conclusion that agrees with Piaget since in Chapter I Piaget was quoted as saying that conservation was necessary for any rational activity.

It seems that the significance of the relationship of conservation to mathematics achievement hinges very closely on the type of achievement test that is used. Nonconservers can probably memorize facts as well as conservers can. Therefore a mathematics achievement test designed to be used in conjunction with a conservation test should have many items which cannot be answered from memory.

The achievement test used in the present study was designed with the above consideration in mind. However, some of the items in the computational section certainly do not meet this criterion, especially for the grade three

subjects. Therefore the _Cooperative Primary Tests -- Mathematics_ was also used because it was designed to be a test of "major concepts of mathematics in their emergent state" (Educational Testing Service, p. 11) and did not ask for simple recall of factual material.

IV. SOME QUALIFYING COMMENTS

It was shown in the previous sections of this chapter that conservation and mathematics achievement are closely related. There are, however, a number of independent variables whose relationship to conservation and/or mathematics achievement tends to qualify the relationship between conservation and achievement in mathematics. Intelligence, socio-economic status, vocabulary level, and listening ability have been singled out for discussion. The first two are considered because of the number of research studies which have found them to be important both in the acquisition of conservation and in mathematics achievement. Research will be reported in this section which indicates that the latter two independent variables may also influence conservation and achievement. However, vocabulary development and listening ability were also included as independent variables in the present study because a large portion of the data collection was done verbally. The subjects were expected to justify their conservation responses. The ability to do this may be affected by a subject's facility

with words. The conservation test and most of the mathematics achievement and mathematics ability tests were administered orally and hence the attentiveness or listening ability of the subjects may have been an important factor in their performance.

Intelligence seems to have a significant influence upon both the acquisition of conservation and mathematics achievement. Overholt (1964) found that there was no significant correlation between conservation of substance and achievement when he statistically controlled for intelligence.

Reimer (1968) found a moderate but significant correlation (0.452) between intelligence and conservation and a correlation of 0.621 between intelligence and mathematics achievement at the grade one level. The correlation between achievement and conservation was 0.412 in Reimer's sample. Since intelligence was significantly correlated to both achievement and conservation, Reimer partialled out the effects of intelligence and obtained a partial correlation of only 0.188 between achievement and conservation. This partial correlation was not statistically significant.

Studies by Lefrançois (1966), Feigenbaum (1963), and Young (1969) found mental age to be related to the acquisition of conservation. Many other studies have found intelligence to be a significant factor in mathematics

achievement. For example, Rose and Rose (1961) found a significant relationship between success in arithmetic and intelligence and Muscio (1962) reported a correlation of 0.766 between quantitative understanding and intelligence quotients.

Some research studies have found socio-economic status not to be a significant factor in conservation or mathematics achievement. On the other hand there are a number of studies which have found it to be an important variable.

Lefrançois (1966) found socio-economic status to be more important than either mental or chronological age in terms of the degree of relationship to the acquisition of conservation.

Wadsworth (1969) was interested in the effect of peer group influence on the learning of conservation of number. He found that the peer group had no significant effect upon the acquisition of conservation but he also observed that:

> ... the mean value of fathers' occupations was higher for conservers than for non-conservers. Also, the mean value of fathers' occupations for students who learned to conserve was higher than for students who did not learn to conserve (p. 4373).

Since fathers' occupation is an important factor in determining socio-economic status, the results obtained by Wadsworth lend some support to the theory that social class

affects the acquisition of conservation.

Rothenberg and Orost (1969) found social class not to be statistically related to the learning of conservation of number. However, the relationship was close to statistical significance and furthermore it was constant over three different experiments. On these grounds Rothenberg and Orost concluded that generally middle class subjects obtain higher pre-test scores on a conservation of number test, middle class experimental (trained in conservation) subjects showed more growth as a result of the training than lower class experimental subjects, and middle class control subjects tended to show some gain from pre- to post-test whereas lower class control subjects tended to decrease during the same period of time. Rothenberg and Orost concluded that in general "the middle-class Ss showed slightly more growth in conservation than the lower-class Ss" (p. 724).

Alkire (1954) compared the functional competence in mathematics of students living in school districts whose assessed value placed them in the upper quarter of all districts with students living in districts in the lower quarter of all districts in terms of assessed value. He found that students in the upper-class districts performed significantly better in mathematics than students in the lower class districts. In an earlier experiment involving only colored subjects, Edmiston and McBain (1945) found the

same type of differences and concluded that economic improvement must improve social status before it will appreciably affect school achievement.

Esther Unkel (1966) studied the interaction of social class and sex factors with the discrepancy between expected mathematics achievement (based on chronological age, grade placement, and scores on a test of mental age) and actual mathematics achievement at the elementary school level. She concluded that:

> . . . in general it can be said that the achievement of children of comparable mental ability is affected by socio-economic status, with pupils in the high socio-economic group attaining the highest achievement level, pupils in the middle group attaining the next level, and students in the low group having the lowest level of achievement (p. 668).

Unkel's findings were supported by the results of a study conducted by Cleveland and Bosworth (1967).

Peters (1969) found language comprehension to be a significant predictor of scores on a conservation of number test. Age and analytic style were also significant predictors of conservation of number. Peters defined language comprehension to be an understanding of comparative words such as more, less, same, different, etc. Peter's study provides some evidence that vocabulary development may influence conservation. Langer (1967), in a paper on the role of vocabulary in concept development, reviews a number of studies which support the hypothesis that the

"development of vocabulary is an inextricable part of concept development" (p. 449). Conservation is also a prerequisite for concept development according to Piaget (1952). Therefore, it is likely that vocabulary and conservation are also correlated and it is possible that vocabularly development could influence the acquisition of conservation.

Language factors also affect achievement in mathematics. Muscio (1962) obtained a correlation of 0.784 between word meaning and quantitative understanding in mathematics. When mental age was partialled out the resulting correlation coefficient (0.471) was still statistically significant. He also found correlations of 0.727 and 0.816 between quantitative understanding and verbal concepts and between quantitative understanding and total language factors respectively. Muscio found that in his sample mental language factors were more closely related to quantitative understanding than were non-language factors.

There is a little evidence that attentiveness (listening ability) is also related to both conservation and achievement. Young (1969) found that attentiveness had a "strong positive correlation with composite conservation score" (p. 3886). She was working with conservation of number in pre-school children.

Lundsteen (1967) investigated the ability of children

to solve problems when the problems were read to the children (problem-listening mode) and when the children were able to read the problems themselves (problem-reading mode). She found a very low correlation between scores on the two modes. A factor analysis indicated that listening was a factor distinct from reading. Lundsteen's finding has implications for the present study since a high proportion of the testing was in the listening mode. The results may have been different if the children would have been required to read all directions and questions.

V. THE PRESENT STUDY IN PERSPECTIVE

To this point the present chapter has shown how the concepts of identity, reversibility, and compensation originate in the theory of Jean Piaget. These concepts seem to play an important role in the acquisition of conservation which in turn seems to affect the achievement level of students in mathematics. Some qualifications were placed upon the latter relationship.

The present study fits into the above scheme by attempting to bridge the gap between the concepts of identity, reversibility, and compensation and mathematics achievement. It seems that identity, reversibility, and compensation affect mathematics achievement indirectly through their role in conservation but we do not know if there is or is not a direct relationship between these concepts and

mathematics achievement. There has been no research which has attempted to establish such a connection. Since there has been no previous research on this problem, the present study should be considered as somewhat of an exploratory approach. If some relationship is found to exist between the concepts of identity, reversibility, and compensation and mathematics achievement then future research should investigate the relationships in a more rigorous manner.

CHAPTER III

INSTRUMENTATION, RESEARCH PROCEDURES, AND ANALYSIS OF THE INSTRUMENTS

I. INSTRUMENTATION

To test the hypotheses stated in Chapter I it was necessary to administer a number of tests. Each instrument is described below.

Conservation Test

The purposes of the conservation test were: (1) to obtain a measure of each subject's ability to conserve and (2) to determine the kinds of rationalizations that students would use for a conservation response.

The conservation test was designed to provide an overall measure of the ability to conserve. It was not intended to be an indication of the ability to conserve some specific property. Four properties were considered in designing the conservation test: number, length, area, and volume. The rationale for a multi-property test was given in Chapter II, on pages 45 and 46.

Although the Piagetian method of questioning is not a standardized technique, the type of question asked to elicit the conservation response is not a significant source of variance. This has been established in recent studies by Pratoomraj and Johnson (1966) and by Mermelstein and

Shulman (1967). However, obtaining the rationalization may be a more sensitive process because it involves a greater degree of verbalization by the child. Therefore a uniform procedure was developed which is described in the next paragraphs.

The question, "How can you tell?" at the end of each item of the conservation test was the stimulus for the child to give his rationalization. If he did not give a conservation response he may or may not have been asked this question. If he did conserve he was, of course, asked the question. Two possibilities existed at this point.

(1) The child gave a rationalization. Some neutral comment such as "I see" was made and then the subject was asked, "Is there any other way that you can tell that they are still the same?" If another response was given, no further responses were asked for. If a second response was not started within approximately ten seconds the experimenter said, "That is fine" and proceeded to the next item.

(2) The child did not respond to the initial question. In this circumstance he was asked other non-directive questions such as, "Why is it that these are still both the same?" or "Why is this one (pointing to the transformed state) the same as this one (pointing to the standard)?" If the child still did not respond the experimenter proceeded to the next item. If a rationalization was given then a second response was requested as in situation

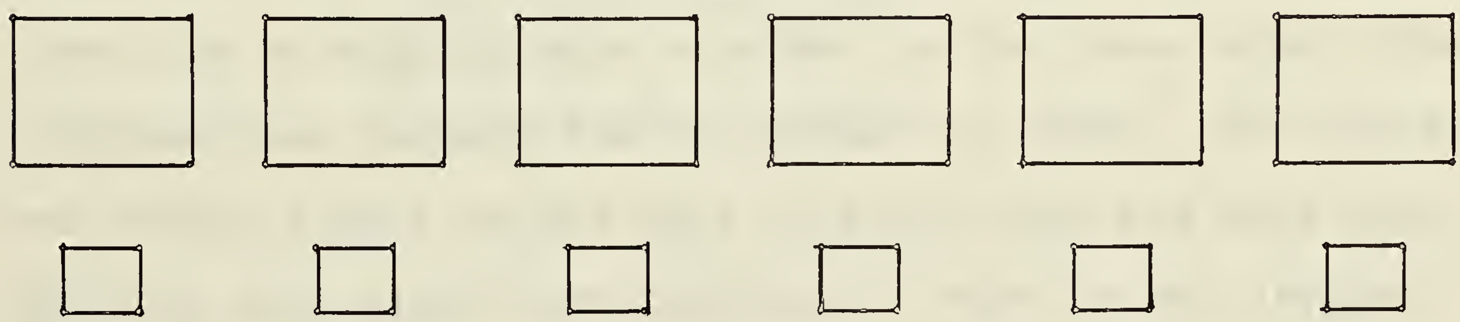
(1). If the subject was hesitant but did respond he was given an encouraging comment such as "Good!" but no further rationalizations were requested.

At no time was a third response asked for although some responses had two or more rationalizations embedded in them.

A subject was never pressed for an answer beyond what was described in situation (2) above.

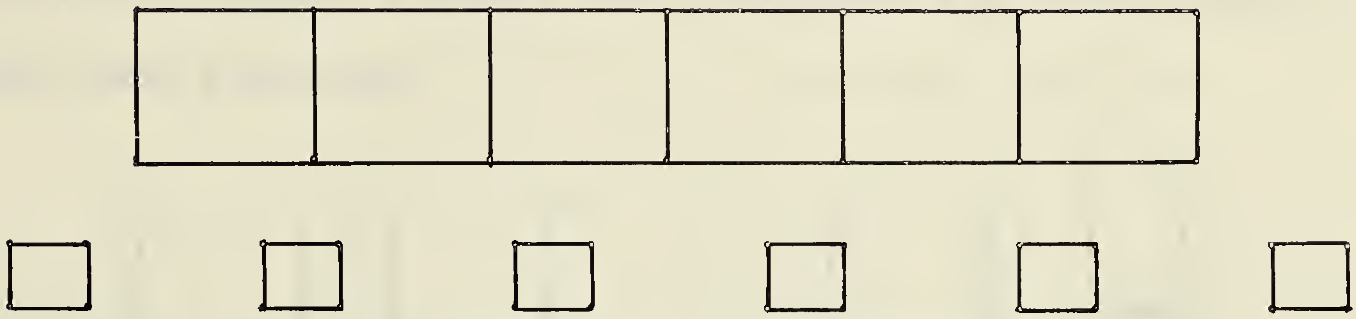
Description of the Conservation Test Items

Item 1. The experimenter gave the subject six half-inch cubes and kept six one-inch cubes. The researcher then put one of his cubes on the desk and asked the subject to put one of his little blocks in front of the big block. This procedure was repeated until all six blocks were on the table in the arrangement shown below.



The child was then asked, "Are there more big blocks or more little blocks or are there the same number of each?" Usually the subject responded that there were the same number. Then he was asked to watch while the researcher pushed the

large cubes together to form this configuration:



The subject was again asked, "Are there more big blocks or more little blocks or are there the same number of each?" If he said that there were still the same number of each his rationalization was elicited as outlined above by asking, "How can you tell that there is still the same number?"

Item 2. The subject was presented with two boxes of beads, one box contained ten red beads, the other box contained ten green beads, and two identical glass containers. There was no significance attached to the color other than to differentiate between the two groups of beads. The subject was asked to pick up one bead of each color and then drop one into each glass simultaneously. When he had dropped all ten beads into the glasses he was asked if there were the same number of beads in each container. If he confirmed this, the experimenter poured the red beads into a glass container with smaller diameter and greater height. Then the subject was asked, "Are there more red beads, or more

green beads, or are there the same number of each?" If he conserved, his rationalization was elicited as described earlier. Figure 2 shows the configuration before and after the pouring.

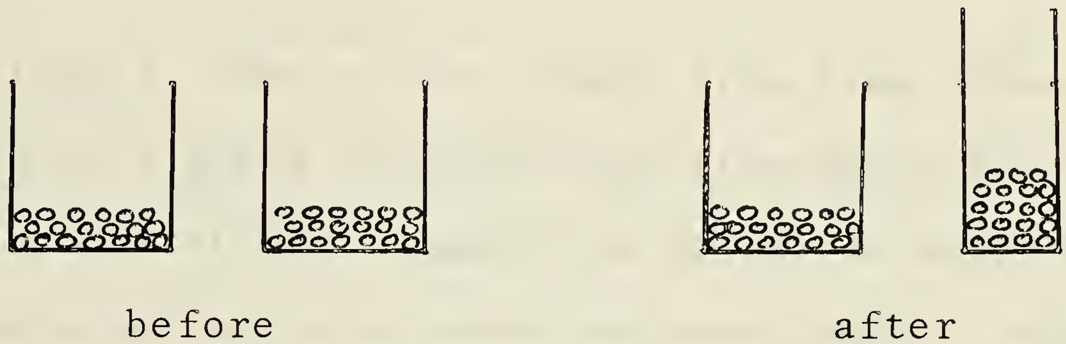
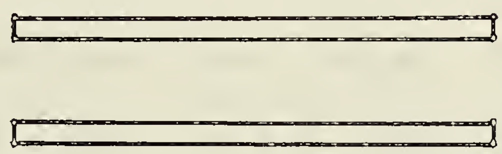


FIGURE 2

CONFIGURATION OF BEADS BEFORE AND AFTER POURING AS DESCRIBED IN ITEM TWO

Item 3. The subject was presented with two sticks about one foot long. The sticks were placed parallel with their ends alinged as shown below.

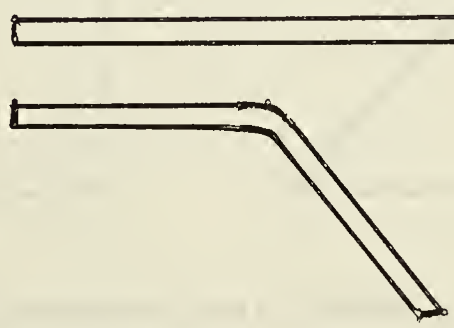


After the subject had confirmed that both the sticks were the same length one of the sticks was moved three or four inches forward but still parallel to the other one as shown below.

The child was then asked, "Now which stick is longer, or are they the same length?"

If the child said they were the same he was asked, "How can you tell that they are the same length?"

Item 4. One of the sticks from item three was replaced by a piece of electrical wire which was the same length as the sticks. Again the child was asked to verify that the stick and wire were the same length. When this was established, the wire was transformed into a curved path with one end remaining stationary as shown in the next diagram.

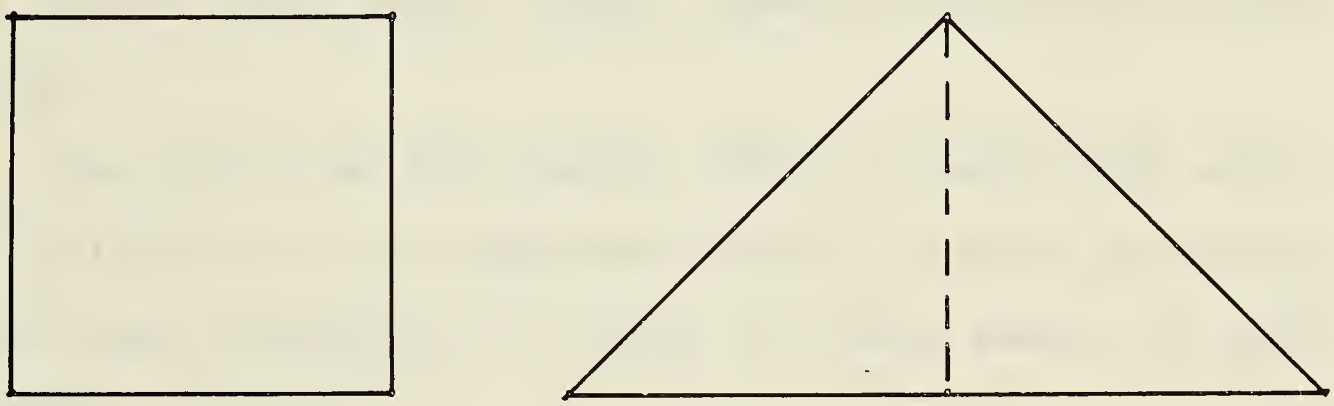


The subject was then asked, "Now which is longer, the stick or the wire, or are they both the same length?"

If the child said that they were the same length he was asked for his rationalization.

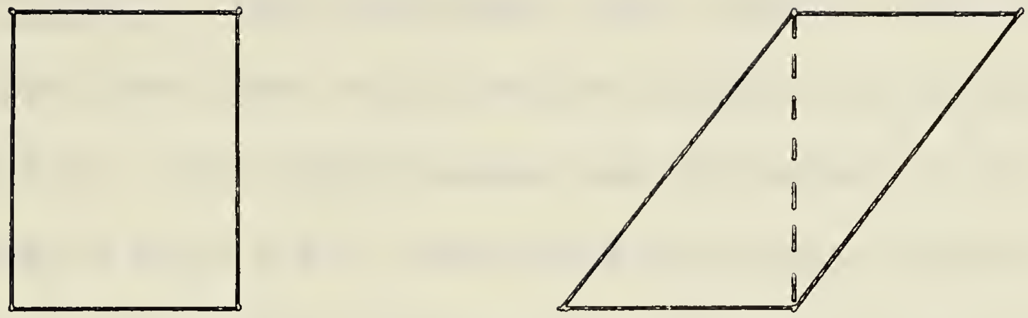
Item 5. Two paper rectangles 4 1/4" by 5 1/2" were placed in front of the subject and he was asked to imagine that these rectangles were two playgrounds, one for him to play on and one for the experimenter to play on. The

experimenter asked, "Is there more room to play on your playground or more room to play on mind, or do we both have the same amount of room to play?" After the subject confirmed that there was the same amount of room he was told that the experimenter did not like the shape of his playground and to watch what he did to it. The experimenter then cut one of the rectangles along its diagonal and flipped one piece over and put the uncut sides together forming a triangle as shown below.



Then the student was asked, "Is there more room to play here or is there more room on this playground or do they both have the same amount of room?" If the subject said that they were the same he was asked, "How can you tell?"

<u>Item</u> <u>6</u>. The same procedure was used as in Item 5 except that the rectangles were 8 1/2" by 5 1/2" and when one was cut it was put together as a parallelogram instead of a triangle. The final configuration is shown in the accompanying diagram.



Item 7. Two identical glass bottles with an equal amount of water in them were placed before the subject. After he had verified that there was the same amount of water in both bottles, the liquid from one was poured into another glass bottle which had a larger diameter than the original bottles.

The child was then asked, "Now is there more water here (pointing to the untouched bottle) or here (pointing to the larger bottle) or is there the same amount of water in both?"

If the subject said there was still the same amount of water in both bottles he was asked, "How can you tell that it is the same?" Figure 3 shows the configuration before and after the water was poured.

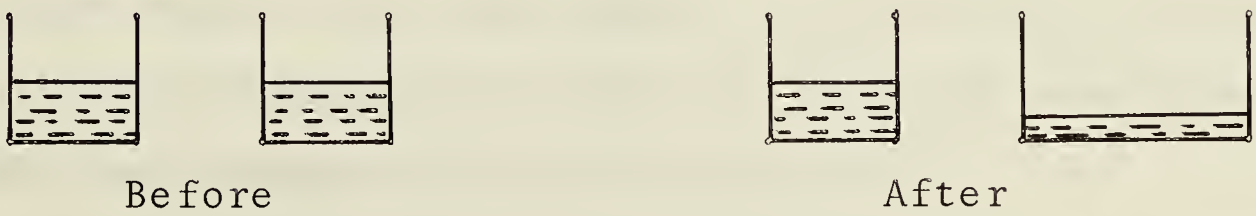
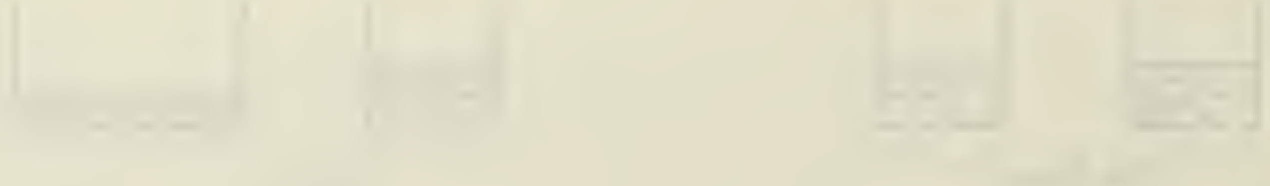
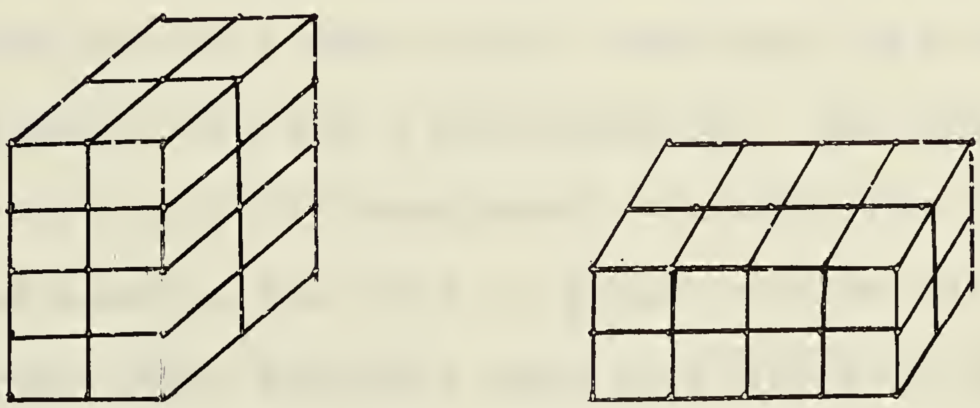


FIGURE 3

PERCEPTUAL CONFIGURATION BEFORE AND AFTER THE WATER WAS POURED



Item 8. Two buildings, one constructed with orange blocks and the other with yellow blocks, were placed before the subject. No significance was attached to the color other than to help the child differentiate between the two buildings. Each building was four blocks high, two blocks wide, and two blocks deep. The student was asked to imagine that these were tall apartment buildings and was asked to verify that both buildings had the same amount of room to live in. Then one of the buildings was transformed by taking the top half and placing it beside the bottom half to form a building two blocks high, four blocks long, and two blocks deep as indicated in the diagram below. Each block was a one-inch cube.



The subject was then asked, "Is there more room to live in this building or in this one or do both buildings have the same amount of room to live in?"

If the subject said that there was still the same amount of room in both buildings he was asked, "How can you tell that it is the same?"

Pilot Study of Conservation Test. A pilot study of the conservation test was conducted in October, 1968 in a single school using five grade two and seven grade three subjects. During the morning each subject was given the test as described in the preceding paragraphs. In the afternoon each subject was given the test again but with item six omitted, and on some of the other items a slightly different transformation was applied. The purpose of this pilot study was three-fold. The first purpose was to determine the adequacy of the test to obtain total and partial conservers at both grade levels as well as a sampling of each mode of rationalization at each grade level.

Two of the five grade two subjects and three of the grade three subjects were total conservers and one subject at each grade level was a nonconserver. The remaining subjects were partial conservers. In addition, each mode of rationalization was used by grade two subjects. None of the grade three subjects used reversibility as a mode of rationalization for conservation. Since the test produced both total and partial conservers and since each anticipated mode of rationalization was given it was decided to use the test as outlined earlier.

The second purpose of the pilot study of the conservation test was to obtain a measure of reliability for the instrument. A test-retest Pearson product-moment

coefficient of correlation was calculated using total test scores with seven out of the eight items included. (Item six was omitted since it was not used in the retest.) The test-retest reliability calculated in the above manner was 0.923 which is significantly different from zero at the .01 level of significance. Therefore, the test was considered to be reliable enough for experimental use.

An item by item test-retest reliability was also calculated to check the reliability of each item. The phi coefficient is the proper statistic to use to find the correlation between items. Ferguson (1966) says that:

> If we assign integers, say, 1 and 0, to represent the two categories of each variable and calculate the product-moment correlation coefficient in the usual way, the result will be identical with (phi) (p. 238).

The procedure suggested by Ferguson was used to calculate the test-retest reliability of each item on the conservation test. The two categories of each variable referred to by Ferguson were pass and fail. The correlation coefficient for each item and the associated probability that the observed coefficient is not different from zero are given in Table I.

It will be noted that each item had a high test-retest reliability except item three. However, since item three was taken directly from Piaget's studies and also since it has been used in many other studies it was decided to retain it in the conservation test for the present study.

TABLE I

TEST-RETEST RELIABILITY OF CONSERVATION TEST ITEMS

Item Number	Correlation Coefficient	Probability that r = 0
1	1.000	.00
2	.775	.01
3	.357	N/S
4	.598	.04
5	.775	.01
7	.775	.01
8	.816	.01

The third purpose of the pilot study of the conservation test was to provide the experimenter with practice in eliciting the rationalizations for conservation by the method described earlier. One outcome of this pilot project was that there seemed to be two different types of identity responses. One type seemed to stress the operational aspect, that is, the transformation was irrelevant. The other type of identity rationalization focused on the objects, that is, the actual materials did not change and therefore must be the same. On the basis of this observation it was decided to define two separate identity modes of rationalization for conservation. These were called "Operational Identity" and "Substantive Identity" respectively. They are more fully defined in this chapter in the section on research procedures.

Mathematics Achievement Test

An achievement test devised by the experimenter was based primarily on the content of the *Seeing Through Arithmetic* (Revised) (Hartung, *et al.*, 1965) texts for grades one and two. For a test to be considered a test of achievement it must be based on the content that the subjects have covered in their school program. All the subjects in the sample for the present study used the *Seeing Through Arithmetic* series.

The tests which accompany the above series were considered to be inadequate because they were designed for use at only one grade level and they tended to measure simple recall. Therefore a special test had to be devised for the present study.

Method of Developing the Mathematics Achievement Test. A first draft of the achievement test was constructed and administered to a sample of sixty-two grade one and two students in early June, 1968. The reliability (Kuder-Richardson Formula 20) of .895 was acceptable and an item analysis revealed only one poor item. A factor analysis using a varimax rotation found that three identifiable factors accounted for about 46 per cent of the total variance. The three factors were identified as knowledge of multiplication and division basic facts, knowledge of addition and subtraction basic facts, and an understanding

of the numeration system. These factors are important at the primary level.

The first draft was used as a foundation for a more extensive achievement test. The revised version consisted of forty-two items. A pilot study of the revised version was conducted in October, 1968. The reliability was increased to .932. After examining each item and its distractors the final version was compiled consisting of thirty-nine items. The final version of the test appears in Appendix A.

Description of Subtests. The Mathematics Achievement Test consisted of four subtests: (1) Geometry; (2) Numeration; (3) Addition and Subtraction Basic Facts; and (4) Multiplication and Division Basic Facts. Table II lists the items on the Mathematics Achievement Test which were used in each of the four subtests.

TABLE II

CLASSIFICATION BY SUBTEST OF ITEMS ON THE MATHEMATICS ACHIEVEMENT TEST

Subtest	Item Numbers	Total Number of Items
Geometry	1, 2, 4, 9, 12, 13, 14, 17, 19	9
Numeration	3, 5, 6, 7, 8, 10, 11, 15, 16, 18	10
Addition and Subtraction	20, 22, 24, 25, 29, 30, 32, 35, 36, 39	10
Multiplication and Division	21, 23, 26, 27, 28 31, 33, 34, 37, 38	10

Method of Administration. The mathematics achievement test was administered as a group test. Adequate instructions were given on the test so that subjects could work through it on their own. However, for items one through nineteen, the experimenter read the instructions or question twice for the students. Time was allowed for all subjects to answer. If some subjects were having trouble the experimenter told them to leave the question and listen to the next one and they would be given time at the end to come back and answer any items that they missed. Items twenty through thirty-nine were done by the subjects at their own speed after the experimenter explained what they were to do. All answers were marked on the test booklet.

Mathematics Ability Test

The Cooperative Primary Tests -- Mathematics, Form 12A was used as a measure of mathematical ability in the current study.

This test was designed by Educational Testing Service, not as a test of number work or computation, but to "test major concepts of mathematics in their emergent state: concepts under the general headings of number, symbolism, operation, function and relation, approximation and estimation, proof, measurement and geometry" (Educational Testing Service, 1967, p. 11).

The test consists of fifty-five items in two separate

parts. In the first part the experimenter reads the stimulus material and the pupils mark the illustration that matches the verbal stimulus. In the second part the student works on his own. A stimulus is presented and he marks an appropriate response. No reading is involved.

Validity of the mathematics ability test must be judged from the content of the test. The items tested a wide variety of concepts which are important concepts for the elementary school mathematics program. The experimenter therefore considered the test to be valid as a test of general mathematics ability.

The reliability of the Cooperative Primary Tests -- Mathematics was computed by Educational Testing Service on the basis of the results obtained from administering the test and an alternate form to three hundred grade two subjects. When the form used in the present study (Form 12A) was administered first the alternate form correlation was 0.84. When the order of administration was changed the correlation between forms was 0.77. The internal consistency coefficient for Form 12A was 0.85 (KR-20) when it was administered first and 0.86 when it was administered second.

Using the same sample the mean biserial correlation between items and total test score was 0.43. This implies that the test discriminates well between individuals of high and low ability where ability is defined as the total test score.

On the basis of the validity, reliability, and item discrimination data the *Cooperative Primary Tests -- Mathematics* Form 12A was judged adequate for the purpose of providing a measure of the general mathematics ability of the subjects in the present study.

Listening Ability Test

The *Cooperative Primary Tests -- Listening*, Form 12A was used to measure the ability of the subjects to listen to verbal instructions. The experimenter read a word, sentence, or short story. The subjects were asked to mark one of three pictures that went best with what the researcher read. In the case of the short stories several questions were asked after each story. Each question required a separate response. Listening, therefore, includes comprehension, recall, and interpretation.

The reliability of the listening test was calculated in two ways. The alternate form correlation was 0.82 when Form 12A was administered first and 0.75 when it was administered second. The internal consistency coefficient (KR-20) of Form 12A was 0.85 when it was administered first and 0.86 when it was administered second.

The mean biserial correlation between items and the total test score was 0.45 which indicates that the test differentiated between good and poor listeners.

Content validity was assumed. The experimenter had

to rely upon the competence of the personnel who designed the test to construct an instrument which measured the ability of a subject to listen.

On the basis of adequate reliability and item discrimination and assuming validity, this test was judged suitable for testing the listening ability of subjects in the present study.

Intelligence Test

The *Raven's Coloured Progressive Matrices,* Sets A, Ab, and B were administered to all subjects as a group test. The results were used as a measure of intelligence in the current study.

The *Coloured Progressive Matrices* do not produce an Intelligence Quotient in the usual sense and, therefore, may not be a test of "general intelligence". The author describes the test as a "test of observation and clear thinking" (Raven, 1956, p. 3). The claim by the author that this test is not a test of general intelligence is somewhat refuted by a study by MacArthur (1960) in which a battery of twelve ability and achievement tests was given to three hundred Edmonton grade three boys. The inter-correlation matrix was factor analyzed (Centroid method) and orthogonally rotated (two-by-two graphical method). Four factors were found to account for about half the total variance. The first factor MacArthur interpreted as a

"general intellectual ability factor similar to Spearman's g" (p. 73). The highest loading on this factor was shared by the Gates Paragraph Reading Test (.559) and the Coloured Progressive Matrices (.556). The Progressive Matrices did not contribute to any of the other factors which MacArthur called "schooling", "verbal", and "number", respectively. MacArthur concluded that the Coloured Progressive Matrices "can be employed as an economical indicator of general intellectual ability ..." (p. 74).

The Coloured Progressive Matrices were designed for children between the ages of five and eleven years as well as for other special groups of people. As the word matrix suggests, the items are two dimensional analogies. A pattern (colored) is presented with a missing portion. The subject is presented with six alternatives for the missing portion, only one of which correctly completes the pattern. The design is altered from left to right according to one principle and from top to bottom according to another principle. However, a few of the items consist of constant patterns. There are twelve items in each of the three sets making a total of thirty-six items. The items progress in difficulty within each set and from set A to set B.

The test was administered to ten subjects at a time. The subjects marked their answers on an answer sheet prepared by the researcher. Directions were carefully given verbally and the first matrix was used as an example.

That is, the subjects were helped with the first matrix so that they would fully understand what was required of them. Each subject then worked the remaining thirty-five matrices by himself and at his own rate of speed. The students were periodically observed to ensure that they were putting their answers in the proper place on the answer sheet.

In addition to the above test, the Intelligence Quotient for each subject was obtained from the cumulative record cards. However, these scores were used as extra data since they were recent for some subjects but as much as two years old for other students and because not all subjects had been given the same intelligence test.

Vocabulary Test

The vocabulary section of the *Wechsler Intelligence Scale for Children* was used as a measure of the subject's vocabulary. It is really a test of the ability to verbalize concepts. This instrument consists of a list of forty words arranged in order of difficulty. Subjects are asked to give a definition for each word beginning with the easiest (bicycle) and continuing until five successive failures have been encountered. A failure is defined as a wrong definition or no definition.

The vocabulary test was administered as prescribed in the manual (Wechsler, 1949, p. 68) except that every subject

was started at word one. The vocabulary test was administered to each subject individually immediately after the conservation test. The definitions were recorded on tape to help with the scoring.

The scoring of the vocabulary test was done as prescribed in Appendix C of the test manual (pages 98 through 109). Each definition was quantified twice, immediately upon its utterance and later when the experimenter listened to the tape. If there was a discrepancy in the two scores the experimenter replayed the tape until he was convinced of the merit of the definition. Each definition was scored 2, 1, or 0 depending on the quality of the definition except words one through five which were scored only 2 or 0. A subject's vocabulary score was his total score on the test.

Socio-Economic Status

Blishen's Occupational Class Scale was used as a measure of socio-economic status in the current study (Blishen, et al., 1968, pp. 741-753). Only the father's occupation was considered except where the father was deceased or otherwise absent in which event the mother's occupation was used.

Blishen's system of determining socio-economic status for occupations was based on the education and income characteristics of incumbents of these occupations. His

first ranking of occupations was based on data obtained from the 1951 Canadian census. The scale was recently revised using the 1961 census data. Blishen reports a rank correlation between the two scales of 0.96. This indicates a stability in the social structure over time and a similarity in results despite slightly different procedures in developing the scale.

In the present study the occupation was taken from the school records and quantified by the use of Blishen's revised scale. Where the occupation listed in the school records was not detailed enough the student was asked to provide more detail. Some difficulty was encountered in a few instances where the occupation listed in the school records did not match any of those listed in Blishen's scale. In some instances the most similar occupation was chosen but in other instances the experimenter was uncertain of the ranking and the occupation was not quantified.

The above procedure has probably introduced some experimenter bias into the data and also lowered the reliability of scoring. This should be considered in interpreting any subsequent results where socio-economic status is a variable.

The correlations among the major variables are presented in Appendix C and the correlations among the mathematics variables are given in Appendix B.

II. RESEARCH PROCEDURES

Sampling Technique

The Edmonton Public School Board was asked to make ten schools available for data collection and the County of Strathcona was asked to supply two schools. Thus the selection of schools was fortuitous and not random.

The principal of each school was then contacted and asked to make one grade two and one grade three class available to the researcher. Therefore, the selection of classes within a school was also fortuitous and not random.

A table of random numbers was used to select five subjects from the grade two class and five subjects from the grade three class within each school. No stratification variables were used other than grade.

Data Collection

The data were collected between mid-November, 1968 and the end of January, 1969.

The conservation test was administered on an individual basis during the morning hours to the ten subjects selected in each school. The items on the test were administered in random order. The complete interview was tape recorded so that student responses could be transcribed for later analysis. No attempt was made to camouflage the recorder. No mention was made of it but if students asked about it they were told that the researcher wanted to listen to them again. The vocabulary

test was given to each subject individually immediately upon completion of the conservation test.

The remaining tests, Mathematics Achievement, Cooperative Primary Tests -- Mathematics and Listening, and the Coloured Progressive Matrices were administered to all ten subjects as a group during the afternoon hours. An attempt was made to alter the order in which these tests were administered. However, the order was affected by the time available before and after recess and it was found that giving the achievement and listening tests before recess and the remaining two after recess was the most efficient procedure. Therefore, any effects due to the ordering of these group tests have not been completely eliminated.

Care was taken to avoid boredom on the part of the subjects. After each test subjects were allowed to go for a drink or to the washroom. The experimenter chatted with the students during this break about many things, all irrelevant to this study, in order to divert the attention of the subjects from the tests for awhile. However, most students indicated that they enjoyed doing the tests and wanted to do more.

The remaining data, age, Intelligence Quotient, and father's occupation, were secured from the cumulative record cards.

<u>Classifying Responses to the Conservation Test</u>

After each item on the conservation test the subjects were asked, "How can you tell?" The responses to this question, if a subject conserved, were classified by the experimenter into one of seven categories.

(1) <u>Operational Identity</u>. This category contained rationalizations based on the argument that no addition or subtraction of the property in question had taken place. Typical responses of this nature include:

"You never took one away."

"You didn't break off any of that part."

Also included in this category were responses to the effect that the transformation applied to the object was irrelevant to the amount of the property. For example:

"You just cut that piece and it still makes it the same."

"You just squeezed them together."

(2) <u>Substantive Identity</u>. Responses placed into this category were those which indicated that the property was invariant because the particular object exhibiting the property had not changed. Typical responses include:

"It was the same paper."

"Because they are the same beads."

"There is still the same amount of blocks."

(3) <u>Reversibility</u>. Responses placed into this category were those which implied that the transformation applied to the object could be reversed and the perceptual configuration returned to its starting status. For example:

"If I moved that one back it would be about level."

(4) <u>Compensation</u>. This category was composed of rationalizations for conservation on the basis that the amount of the property in one dimension of the configuration was compensated for by an equal amount in another dimension. Some typical responses include the following:

"This is higher and this is lower and this is lower because its a wider jar."

"This one is more over here and this one is more over here."

(5) <u>Other Rational Responses</u>. This category included rationalizations that were logical but could not be placed into one of the other four categories. The response, "I could count them" would fit into this category. Another common type of response in this category was related to reversibility but could not be classed as such. It was a modelling type of response in which the subject said that he could transform the standard to conform to the stimulus instead of changing the stimulus to match the standard as in reversibility. For example, given the two sticks with B

moved forward as in Figure 4 (a), the subject would say that A and B were the same length because he could move A so that its endpoints coincided with B as in Figure 4 (b).

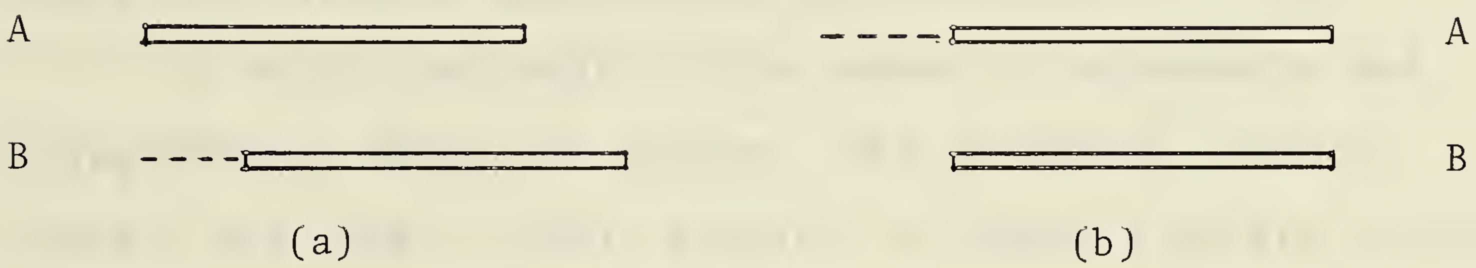


FIGURE 4

ARRANGEMENT BEFORE AND AFTER A MODELLING RESPONSE
(Dotted line indicates path of movement.)

(6) Non-Classifiable Responses. This category consisted of irrational responses and responses which were not interpretable. No use was made of this category in the analysis of data.

(7) No Response. A record was also kept of subjects who conserved but could not or would not give a rationalization for their conservation response. There were twelve subjects who did not respond on one item and five subjects who did not respond to two items. No use was made of this information in subsequent analysis.

Reliability of Response Classification. To check the reliability of the experimenter's classification of student rationalizations a random sample of eleven student responses was printed along with a description of the conservation test

item to which the rationalization was given. These selected responses were given to a panel of seven judges who were asked to classify independently the responses using the category definitions given above.

A record was kept of the number of agreements and disagreements among all judges. The Arrington formula (Feifel and Lorge, 1950), 2a/(2a + d) where a refers to the number of agreements and d to the number of disagreements, was used as a measure of interjudge reliability. This technique had to be used since the correlation and analysis of variance methods of getting interjudge reliability were not appropriate for the nominal data available in the present study.

The seven judges had a total of one hundred and forty-nine agreements and sixty disagreements over all judges and over all eleven responses. These data resulted in an interjudge reliability of .832 using the above formula. When the researcher was included as an eighth judge the reliability was reduced to .823.

The reliability of categorization was also calculated using each judge and the experimenter. The judge-experimenter reliabilities are given in Table III. These reliabilities were also computed using the Arrington formula.

TABLE III

RELIABILITY OF CATEGORIZATION OF RATIONALIZATIONS BETWEEN JUDGES AND THE EXPERIMENTER

Judge	Agreements	Disagreements	Arrington Reliability
1	5	6	.625
2	7	4	.778
3	8	0	1.000
4	6	5	.706
5	8	3	.842
6	6	4	.750
7	8	3	.842

Quantification of Rationalizations

Each rationalization that a subject gave to each item on the conservation test was assigned a weight of three if it was the first rationalization to that item, a weight of two if it was the second rationalization to that item and a weight of one if it was the third rationalization to the same conservation test item. This procedure was agreed upon by a group of advisors on the assumption that the first rationalization given was the one most dominant in the thinking of the subject and should therefore be weighted heavier than subsequent rationalizations to the same conservation test item.

A subject was placed into the rationalization category for which he received the highest total score

over the whole conservation test. Subjects who received equal scores on two or more categories were eliminated from most of the analyses as were nonconservers and subjects in categories five, six, and seven. This elimination left ninety-five subjects distributed into the first four categories of rationalization, Operational Identity, Substantive Identity, Reversibility, and Compensation.

III. EVALUATION OF THE INSTRUMENTS

The two preceding sections of this chapter contained a description of each instrument that was used in the current study and a discussion of how the instrument was used. This section reports a post-administration analysis of the instruments using the data from the present study.

Conservation Test

The mean score for the total sample on the conservation test was 5.375 and the standard deviation was 2.665. The grade two mean was 4.88 compared to 5.87 for grade three. The respective standard deviations were 2.88 and 2.33.

Using a one-tailed t-test it was found that the grade two mean was significantly different from the grade three mean at the .02 level of confidence.

The conservation test had a KR-20 reliability of 0.771 for the total sample. At the grade two level the

KR-20 formula produced a reliability coefficient of 0.791 and for grade three it was 0.750.

It was concluded from an item analysis of the test that all eight items functioned well using the ability to conserve as the criterion. The biserial correlations ran from 0.94 to 1.0 for the total sample. They ran from 0.90 to 1.1 for grade two and from 0.86 to 1.1 for grade three. The biserial correlation is a measure of the ability of each item to discriminate between low scorers and high scorers on the total test. It is essentially the correlation between passing or failing an item and the total test score. Table IV contains the biserial correlation for each item and the proportion of subjects that conserved on each of the items.

TABLE IV

BISERIAL CORRELATION* AND DIFFICULTY INDEX FOR EACH ITEM ON THE CONSERVATION TEST

Item Number	Grade Two		Grade Three		Total Sample	
	Biserial	Diff.	Biserial	Diff.	Biserial	Diff.
1	.903	.667	1.035	.850	.961	.758
2	.957	.783	.863	.900	.946	.842
3	1.056	.517	.972	.567	1.006	.542
4	.995	.467	.931	.500	.951	.483
5	.967	.583	.957	.700	.964	.642
6	.993	.600	.959	.767	.986	.683
7	.977	.633	.866	.767	.940	.700
8	.853	.633	1.117	.817	.959	.725

*Some of the biserial correlations are distorted due to the gross departure from .5 of some of the difficulty indices.

It will be noted that the order of difficulty does not conform to the order given by Piaget (1960). He claims that the order of difficulty should be number, length, area, volume. However, the data presented in Table IV cannot be used to contradict Piaget since there are only two items dealing with each of the four concepts. Furthermore, the water item (item seven) is not measuring the ability to conserve volume as Piaget defines it but rather quantity which should be attained at about the same time as number. Item eight is, however, an example of a volume item but it appears to be rather easy for the subjects used in this study.

Figure 5 shows the distribution of scores on the conservation test for grade two, grade three, and for the total sample. Obviously this distribution is not normal. This is according to design as one of the purposes of the conservation test was to get as many conservers as possible in order to get a better sampling of the rationalizations children give for conservation.

On the basis of face validity and adequate reliability (.77) and on the basis of the item analysis reported earlier it seems that the conservation test designed for this study was a useful and adequate instrument.

Mathematics Achievement Test

The mathematics achievement test mean and variance for

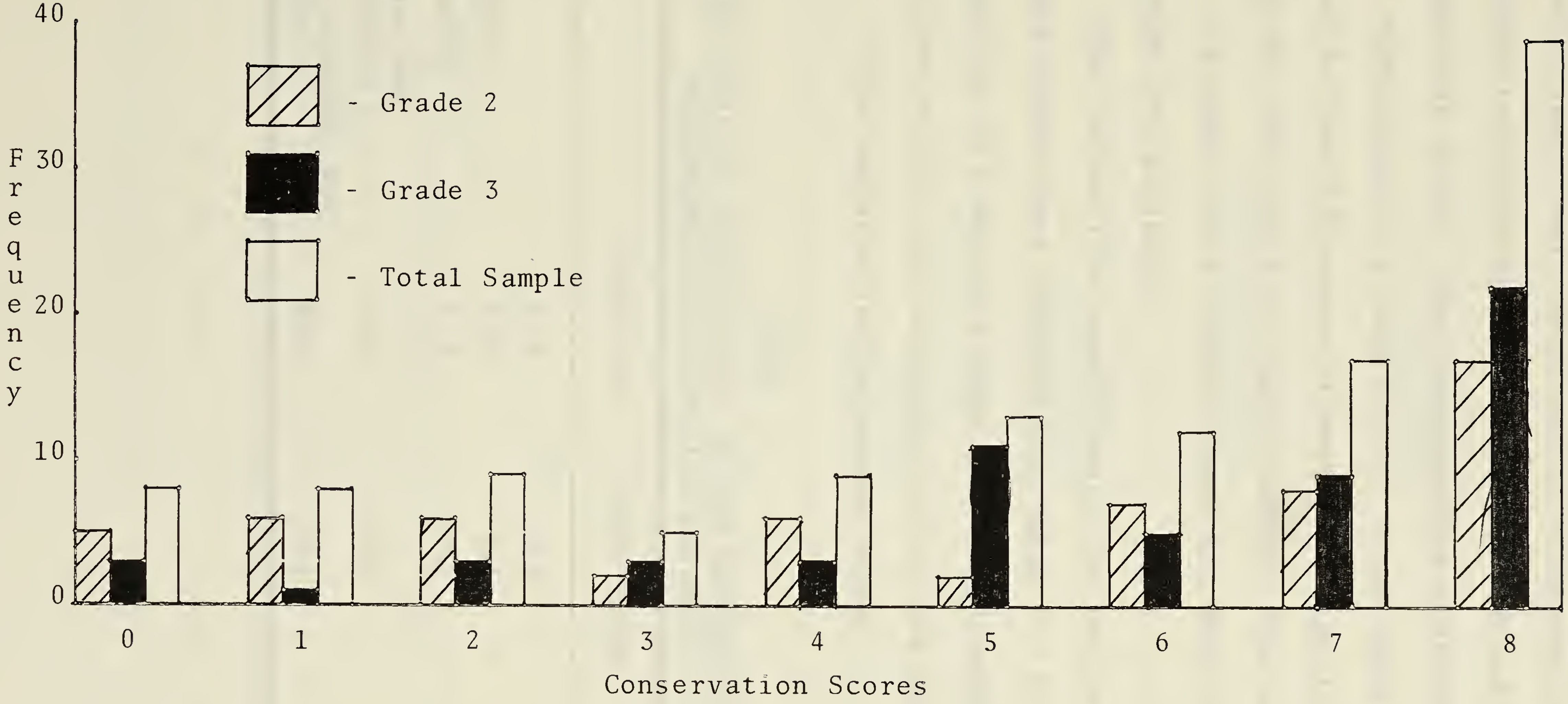


FIGURE 5

DISTRIBUTION OF SCORES ON THE CONSERVATION TEST

the total sample were 24.16 and 62.43 respectively. For the grade two subjects the mean was 19.48 and the variance was 41.65. The corresponding statistics for grade three were 28.83 and 39.51. A one-tailed t-test applied to the difference of means was significant at beyond the .001 level of significance. This was expected since the grade three subjects have had more schooling than the grade two subjects.

The reliability coefficients for the total test and for each subtest are presented in Table V. A total test reliability of 0.89 was judged to be high enough to accept the scores on the test as indicative of the achievement level of the subjects in the present study.

TABLE V

RELIABILITY COEFFICIENTS OF THE MATHEMATICS ACHIEVEMENT TEST AND ITS SUBTESTS

Test	Grade Two	Grade Three	Total Sample
Total	.857	.863	.893
Geometry	.610	.461	.574
Numeration	.473	.661	.655
Addition and Subtraction	.748	.693	.770
Multiplication and Division	.906	.814	.919

The distribution of scores on the mathematics achievement test is shown in Figure 6. The distribution was found to be normal by the chi-square goodness of fit test. The skewness was -0.127 and the kurtosis was -0.934.

The biserial correlations comparing each item to the total test ran from a low of .179 on item fifteen to a high of .982 on item thirty-seven. The difficulty index reached a low of .133 on item fifteen and a high of .983 on item twenty-five. The mean biserial correlation was .631 which indicates that the test discriminated well between low and high achievers as determined by the total test score. Most of the distractors seemed to work well.

On the basis of the reported reliability, item discrimination, distribution of scores, and on the basis of face validity it was concluded that the Mathematics Achievement Test was an acceptable instrument for measuring the achievement in mathematics of grade two and grade three students.

Other Instruments

The *Cooperative Primary Tests*, *Raven's Coloured Progressive Matrices*, and the vocabulary section of the *Wechsler Intelligence Scale for Children* were standardized instruments. It was therefore concluded that no special analysis of them was needed. Table VI gives some statistics for these instruments.

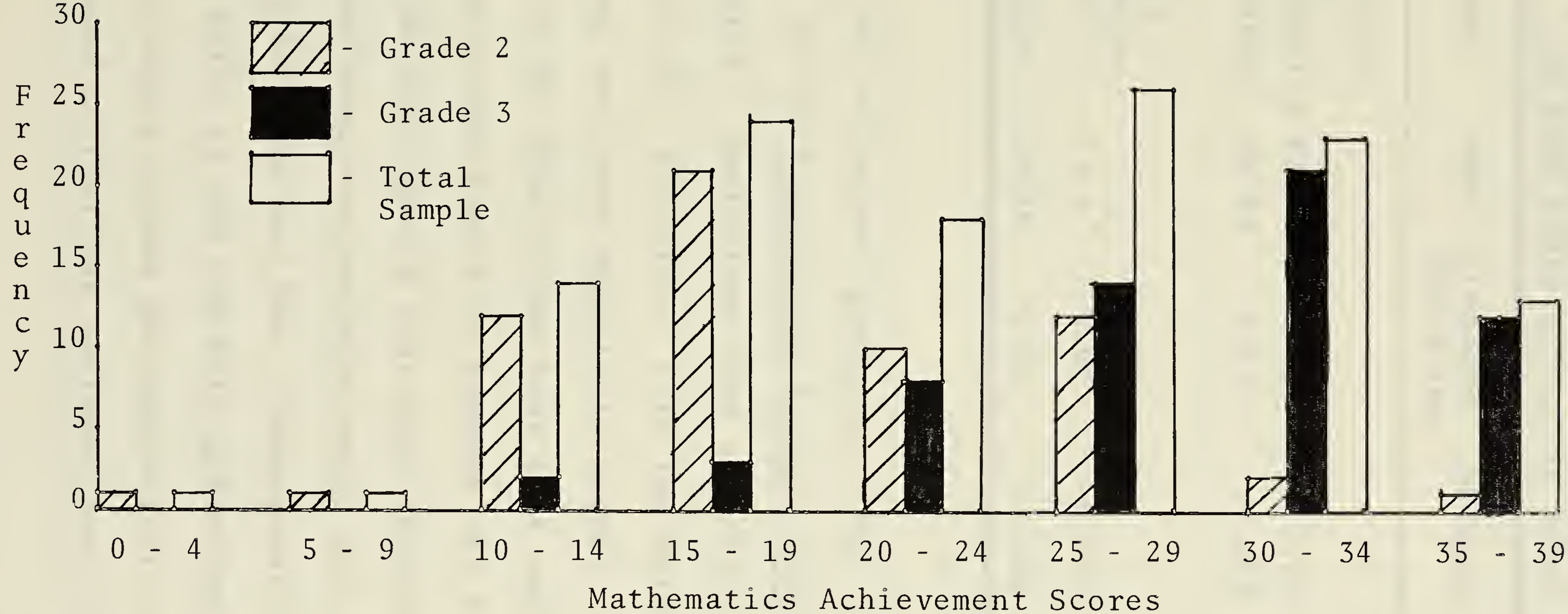


FIGURE 6

DISTRIBUTION OF SCORES ON THE MATHEMATICS ACHIEVEMENT TEST

TABLE VI

MEAN, STANDARD DEVIATION, SKEWNESS, AND KURTOSIS ON FOUR TESTS USING THE TOTAL SAMPLE

Instrument	Mean	Standard Deviation	Skewness	Kurtosis	Prob. of χ^2
Cooperative Primary					
Mathematics	42.42	8.59	-1.525	3.700	.006
Listening	38.84	5.65	-1.218	2.211	.026
Ravens Coloured Progressive Matrices	23.37	6.66	-0.632	0.205	.130
Vocabulary	24.16	6.88	-0.344	-0.165	.327

The chi-square goodness of fit test indicated that neither of the Cooperative Primary Tests was normally distributed. Deviations from normality were significant at beyond the .05 level. Both the mathematics and the listening test were negatively skewed. This is likely a result of the form of the test that was used. These tests had to be ordered well in advance and at the time of ordering it was anticipated that data collection would take place in October when the subjects would only be one to two months out of grades one and two. Consequently Form 12A was ordered which is for use with grades one and two students. However, delays beyond the control of the author were encountered so that the subjects were three to five months

out of grades one and two when the tests were administered. This time and grade factor may account for the high scores on the mathematics ability and listening tests.

CHAPTER IV

RESULTS OF THE INVESTIGATION

This chapter reports the results of testing the hypotheses and other tests made on the data. Most of the analyses reported in this chapter were carried out on the IBM 360/67 computer using programs that have been carefully tested by the Division of Educational Research Services of the University of Alberta. The remainder of the analyses reported were carried out by the use of APL (A Programming Language) functions written by the researcher and carefully tested with data for which the results were known. The latter method made use of the same computer.

The results are reported separately for each of the three major purposes stated in Chapter I. The hypotheses listed under each purpose in the first chapter are discussed in turn.

I. THE USE OF DIFFERENT MODES OF RATIONALIZATION

Hypothesis One

Hypothesis One Restated. There is no significant difference in the observed frequency with which each mode of rationalization is chosen and a rectangular frequency distribution expected by chance for:

(a) partial conservers,

(b) total conservers, and for

(c) all subjects,

<u>Test Procedure</u>. Hypothesis One was tested by a Kolmogorov-Smirnov one-sample test which compares the observed frequency distribution with an equiprobable distribution expected by chance. The test focuses on the largest deviation between the observed cumulative proportional frequency and the theoretical cumulative proportional frequency. The Kolmogorov-Smirnov test was used because Siegel (1956) claims that it is a more powerful test than the chi-square test.

<u>Results</u>. The frequency distribution for each of the four modes of rationalization is given in Table VII for the total sample, and for total conservers and partial conservers separately.

The maximum deviation of the distribution for the total sample from a chance distribution was 0.300 which was significant at beyond the .01 level. (The .01 level of significance required a deviation of at least 0.167 for N = 95.) Therefore the distribution obtained is not likely a chance distribution. This implies that the subjects used in this study seem to have a preferred mode of rationalization, namely identity.

Grade level does not seem to have any effect upon the foregoing result because when the grade two and the

TABLE VII

FREQUENCY OF THE USE OF THE DIFFERENT MODES OF RATIONALIZATION BY ALL SUBJECTS AND BY TOTAL AND PARTIAL CONSERVERS

Rationalization	Frequency								
	Total Sample			Total Conservers			Partial Conservers		
	Total	Gr. 2	Gr. 3	Total	Gr. 2	Gr. 3	Total	Gr. 2	Gr. 3
Operational Identity	42	17	25	28	11	17	14	6	8
Substantive-Identity	34	16	18	15	8	7	19	8	11
Reversibility	9	5	4	7	4	3	2	1	1
Compensation	10	5	5	3	1	2	7	4	3

grade three distributions were tested separately, both were found to be significantly different from a chance distribution. The maximum deviation from rectangularity for the grade two subjects was 0.267 (N = 43) and for grade three it was 0.327 (N = 52).

When only total conservers were used in the analysis, the same result occurred. The maximum deviation from a rectangular distribution was 0.311 which was significant at beyond the .01 level for a sample size of fifty-three. Again the grade level had no effect upon this result as the maximum deviation of 0.292 for the twenty-four grade two total conservers was significant at the .05 level and the maximum deviation from rectangularity of 0.337 for the twenty-nine grade three subjects was significant at the .01 level.

On the basis of the results reported in the preceding paragraph it appears that total conservers also have a preferred mode of rationalization. From the data in Table VII for total conservers it appears that this preferred mode is identity.

The distribution of modes of rationalization among partial conservers had a maximum deviation from rectangularity of 0.286 which was significant at beyond the .01 level for a sample of forty-two subjects. This result held for the distribution of responses by the grade three partial conservers (maximum deviation = 0.326; N = 23) but the maximum deviation of 0.237 for the nineteen grade two

partial conservers was not significantly different from a chance distribution.

From Table VII it appears that the partial conservers also prefer to give the identity rationalizations for conservation.

Conclusion. The above analyses seem to indicate that primary school students do prefer to rationalize conservation with identity-type arguments. At least the identity rationalizations were preferred as a first response. This was true for total conservers and for partial conservers as well as for the total sample. Therefore all three parts of Hypothesis One must be rejected. Grade level had no significant effect upon these results.

Hypothesis Two

Hypothesis Two Restated. There is no significant relationship between the types of conservers (total or partial) and the mode of rationalization expressed for conservation.

Test Procedure. Hypothesis Two was tested by means of a chi-square test of independence. This test was used instead of a Kolmogorov-Smirnov two-sample test because it was desired to subdivide the data to check the effects of other variables. This subdivision was easily carried

out with existing computer programs when the chi-square test was used.

Results. Table VIII contains the data on which the chi-square tests were performed.

A chi-square as large or larger than the 8.35 with three degrees of freedom observed in the analysis for the total sample of ninety-five subjects could occur only about four per cent of the time by chance. On the basis of this result it appears that there is a significant relationship between the types of conservers and the mode of rationalization used for conservation.

The percentages in Table VIII are helpful in trying to describe the nature of the above relationship. It appears that while both groups used the identity modes most frequently, partial conservers tended to use compensation more than total conservers who in turn used reversibility arguments to a greater extent than partial conservers. Furthermore, in terms of the two identity categories a greater proportion of partial conservers were placed in the Substantive Identity category than into the Operational Identity category. The opposite was true of total conservers.

The same relationship seems to hold for low socio-economic status students but not for higher social class subjects. This conclusion was based on the data and chi-square statistics reported in Table VIII.

TABLE VIII

COMPARISON OF TOTAL AND PARTIAL CONSERVERS ON MODE OF RATIONALIZATION FOR THE TOTAL SAMPLE AND FOR SOCIO-ECONOMIC STATUS GROUPS

Mode	Total Sample				Low SES				High SES			
	Total		Partial		Total		Partial		Total		Partial	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
Operational Identity	28	52.8	14	33.3	17	56.7	6	26.1	9	42.9	7	41.2
Substantive Identity	15	28.3	19	45.2	7	23.3	11	47.8	8	38.1	7	41.2
Reversibility	7	13.2	2	4.8	4	13.3	0	0.0	3	14.3	2	11.8
Compensation	3	5.7	7	16.7	2	6.7	6	26.1	1	4.8	1	5.9
Chi-Square	8.353				11.425				0.097			
Probability of chi-square	.039				.010				.992			

In addition to socio-economic status, the data were also subdivided on the basis of the two grade levels, sex, three achievement levels, three intelligence levels, three mathematical ability levels, three listening levels, three vocabulary levels, two age groups, and into two groups defined on the basis of the number of items on the conservation test to which they gave more than one rationalization. The categories for the latter were two through four items and five or more items. The other two-category variables except sex were subdivided with the means as the cutting point. The three-level categories were established by using cutting points at plus and minus one-half standard deviation from the mean.

Subdividing the data in the above manner and then performing a chi-square test for each level of each independent variable provides a type of "control" for the independent variable. When each level of each independent variable listed in the preceding paragraph was "controlled for" one at a time by using the subdivision technique no significant relationship was found between the types of conservers and the mode of rationalization expressed for conservation.

Conclusion. Considering only the total sample there was a significant relationship between mode of rationalization and the type of conserver for the subjects used in the

present study. On this basis Hypothesis Two would have to be rejected. The statistical significance of the relationship was retained for students of low socio-economic status but was lost when the data were subdivided by sex, grade, mathematics achievement, mathematical ability, intelligence, listening ability, vocabulary, age, and number of rationalizations. Furthermore, when the data were tested with the Kolmogorov-Smirnov two-sample test, the difference between the two distributions was found not to be statistically significant.

Hypothesis Three

Hypothesis Three Restated. The mode of rationalization for conservation is independent of the degree of conservation. Degree of conservation was determined by the number of conservation responses made.

Test Procedure. Hypothesis Three was tested by the use of the chi-square test of independence.

Hypothesis Three is related to Hypothesis Two in that conservation scores are compared to the modes of rationalization. However, Hypothesis Three allows for a finer categorization of conservation scores. This was accomplished by blocking conservation scores in two ways: (1) two or three, four through six, and seven or eight, and (2) two through four, five or six, and seven or eight.

Results. A summary of the data for both blocking methods is contained in Table IX.

TABLE IX

RELATIONSHIP BETWEEN MODE OF RATIONALIZATION AND CONSERVATION SCORES BLOCKED BY TWO METHODS

Mode of Rationalization	Conservation Scores					
	Blocking Method 1			Blocking Method 2		
	2-3	4-6	7-8	2-4	5-6	7-8
Operational Identity	1	13	28	5	9	28
Substantive Identity	5	14	15	9	10	15
Reversibility	0	2	7	0	2	7
Compensation	3	4	3	4	3	3
Chi-Square		13.270			10.450	
Probability of chi-square		.039			.107	

There may be a slight distortion to the value of chi-square due to the low frequencies in some of the cells. However, on the basis of Hays (1963) discussion of this problem the distortion is probably not large enough to change the significance aspect of the probability of chi-square.

Using blocking method one there was a significant relationship between the type of rationalization a student used for conservation and the score he made on the conservation test. Examining the percentages by columns

associated with the data in Table IX (blocking method one) it seems that the poor conservers (scores of two or three) used substantive identity and compensation more than the total conservers (55.56 per cent compared to 28.3 per cent for operational identity and 33.33 per cent compared to 5.66 per cent for compensation) and the total conservers used operational identity and reversibility to a greater extent than the poor conservers. Nearly 53 per cent of the total conservers used operational identity and 13.21 per cent used reversibility compared to 11.11 per cent and zero per cent respectively for the poor conservers.

The significance of the relationship between modes of rationalization and conservation scores held true when the data were analyzed separately for male subjects (probability of chi-square = .03), low socio-economic status students (probability of chi-square = .03), and for subjects who had low scores on the listening test (probability of chi-square = .05). Other subdivisions of the data resulted in no significant relationship between mode of rationalization and conservation scores.

The same type of relationship seemed to hold when the conservation scores were grouped by method two. (See Table IX.) However, the relationship was no longer statistically significant and all subdivisions of the data on the basis of other variables resulted in no significant relationships between mode of rationalization and conservation.

Conclusion. It was not possible either to accept or to reject Hypothesis Three since an alteration in the cutting points applied to the conservation scores produced results that were in one instance significant and in the other non-significant.

Hypothesis Four

Hypothesis Four Restated. The mode of rationalization is independent of the difficulty of the items on the conservation test.

Test Procedure. The chi-square test was used to test the relationship between item difficulty and mode of rationalization. In this analysis each response to each item was considered in formulating the two-way classification.

Results. Items one and two on the conservation test were classified as easy items since the mean proportion of subjects passing these items was 0.80. Items five, six, seven, and eight were classified as of moderate difficulty. The mean proportion passing these items was 0.69. The most difficult items were items three and four. The mean proportion of students passing these two items was 0.51.

Table X summarizes the data related to Hypothesis Four.

Nearly eighty per cent of the responses to the easy items were in the identity modes, about sixty-seven per cent of the responses to the moderately difficult items were in

TABLE X

RELATIONSHIP BETWEEN DIFFICULTY LEVEL OF THE CONSERVATION TEST ITEMS AND THE MODE OF RESPONSE

Difficulty Level	Mode of Rationalization			
	Operational Identity	Substantive Identity	Revers-ibility	Comp-ensation
Easy	86	86	16	29
Moderate	142	134	44	91
Difficult	74	29	24	11

Chi-square = 40.91 Probability of chi-square $< .001$

the identity modes, and about seventy-five per cent of the responses on the two most difficult items were identity responses. It also appears that the compensation mode was used to a greater extent than reversibility on easy and moderate items but reversibility was used more than compensation when the items became difficult.

The above relationships between item difficulty and mode of rationalization held within each grade and for both sexes. The data was not subdivided by any other variables.

The classification of items by difficulty corresponds very closely to their classification by topic. Therefore a chi-square test was made of the relationship between topic or property conserved and the mode of rationalization. Table XI contains the data to which the chi-square test was applied.

Examining the data in Table XI by columns it is apparent that the subjects in the present study used mainly

TABLE XI

RELATIONSHIP BETWEEN CONSERVATION PROPERTY AND MODE OF RATIONALIZATION

Mode of Rationalization	Topic			
	Number	Length	Area	Volume
Operational Identity	86	74	91	51
Substantive Identity	86	29	61	73
Reversibility	16	24	23	21
Compensation	29	11	4	87

Chi-square = 137.27 Probability of chi-square $<.001$

identity arguments to rationalize conservation of number with compensation being the next most used category and reversibility the least used. The same is true on the items dealing with area except that reversibility and compensation change places. The use of operational identity to justify conservation of length far exceeded the other categories. However, on the two items defined as conservation of volume, subjects resorted to compensatory arguments to justify the invariance of the property. The same relationships held for both grade levels and for both sexes.

Since every response to every item was used in tabulating the data for Tables X and XI it is highly unlikely that the events are independent. The violation of the independence assumption should be kept in mind when interpreting the significance of the above relationships.

Conclusion. The null hypothesis that there was no relationship between item difficulty and mode of response was rejected. There was a significant relationship between the difficulty of a conservation test item and the type of rationalization a subject used to justify conservation.

II. DIFFERENCES IN THE CHARACTERISTICS OF STUDENTS WHO USE DIFFERENT MODES OF RATIONALIZATION

Hypothesis Five

Hypothesis Five Restated. There is no significant difference between students exhibiting different modes of rationalization for conservation on the following criteria:

(a) intelligence,

(b) socio-economic status,

(c) vocabulary,

(d) listening ability, and

(e) conservation.

Test Procedure. Hypothesis Five was tested by a one-way analysis of variance with the Newman-Keuls comparison of ordered means used as an *a posteriori* test of differences between all four rationalization groups. The computer program used in this analysis (ANOV15) performed both a Newman-Keuls and a Scheffe test on the group means. The Newman-Keuls test was used because it is less rigorous

than the Scheffe test. Since the relationships investigated in the present study have not been examined before it was decided that the less rigorous technique was appropriate. However, the Newman-Keuls test was rendered somewhat more conservative in the present study by the fact that there was a large disparity in the number of subjects in each group. The Newman-Keuls test is partially based on the mathematical term $\sqrt{MS_e / \tilde{n}}$ where the divisor, $\tilde{n}$, is the harmonic mean of the number of subjects in the groups. With a wide disparity in the group sizes the harmonic mean becomes smaller and hence the above term becomes larger. This results in a larger difference between means needed for significance than would have occurred with equal sample sizes.

Table XII contains the means and standard deviations for the four rationalization groups as well as the values of F obtained from the analysis of variance and the probability of obtaining an F as large or larger than the observed. The probability level of the observed F is based on three degrees of freedom in the numerator and 91 degrees of freedom in the denominator for all the analyses except where socio-economic status was the criterion. In the latter instance there were three and eighty-seven degrees of freedom in the numerator and denominator respectively.

In all the analyses the variance was homogeneous. This was determined by the chi-square goodness of fit test.

TABLE XII

COMPARISON OF THE FOUR RATIONALIZATION GROUPS ON INTELLIGENCE, SOCIO-ECONOMIC STATUS, VOCABULARY, LISTENING ABILITY, AND CONSERVATION

Criterion	Operational Identity		Substantive Identity		Reversibility		Compensation		F	Prob. of F
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.		
Intelligence	23.50	5.86	23.68	7.61	28.00	3.61	23.50	6.52	1.28	.286
Socio-Economic Status	45.81	16.15	49.07	16.80	54.56	19.94	39.53	10.35	1.59	.197
Vocabulary	25.10	6.81	25.24	6.07	28.56	5.94	24.00	6.77	0.91	.442
Listening	39.93	4.05	39.94	5.48	38.56	6.84	39.10	4.07	0.27	.847
Conservation	6.71	1.63	5.79	1.98	7.44	1.13	5.20	1.87	4.31	.007

Results. (a) Intelligence. Intelligence was defined as the total score obtained on the Coloured Progressive Matrices. From Table XII it is apparent that the reversibility group had the highest mean score on the above test. However, when the data were subjected to the one-way analysis of variance the probability of getting an F value as large or larger than the 1.28 obtained was found to be 0.286. In other words, the differences in means among the groups could easily be due to chance and hence the differences are not significant.

(b) Socio-Economic Status. Subjects in the reversibility group had the highest mean score on Blishen's Occupational Class Scale. An analysis of variance yielded an F of 1.59 which had an associated probability of 0.197. This result indicates that there were no significant differences in socio-economic status among the four rationalization groups.

(c) Vocabulary Development. The same general results were obtained when vocabulary scores were the criterion as when intelligence and socio-economic status were the criteria. That is, the subjects who used mainly reversibility as a mode of rationalization for conservation had the highest mean vocabulary score but the differences among the four groups were not significant.

(d) Listening Ability. There were no significant differences among the mode of rationalization groups on the

listening test. The means reported in Table XII are very close to each other. An F of only 0.27 was observed in the analysis of variance. An F of 0.27 could occur about 85 per cent of the time just by chance.

(e) Conservation. The ability to conserve was significantly different among the four groups at beyond the .01 level. Applying the Newman-Keuls test to the mean scores contained in Table XII the differences appear to be due to the differences between the compensation group and the reversibility group and between the substantive identity group and the reversibility group. In both instances the students who rationalized with reversibility arguments were the better conservers. However, they were not significantly superior to the operational identity group.

Conclusion. On the basis of these results, parts (a) through (d) of Hypothesis Five were accepted. There were no significant differences among students who used different modes of rationalization for conservation in intelligence, socio-economic status, vocabulary scores, and listening ability.

Part (e) of Hypothesis Five was rejected. There were significant differences among the rationalization groups on their ability to conserve. Students who used arguments considered representative of the reversibility category seemed to be the best conservers. They had a mean of 7.44

out of a possible 8.00. However, this was also the smallest group. It had only nine subjects.

Hypothesis Six

<u>Hypothesis Six Restated</u>. There is no significant relationship between mode of rationalization for conservation and:

(a) age,

(b) grade, and

(c) sex.

<u>Test Procedure</u>. Hypothesis Six was tested with a chi-square test of independence. Two age groups were formed by putting all subjects who were younger than the mean age of 96.5 months into one group and those who were at or above the mean age into the second group.

<u>Results</u>. There was no significant relationship between mode of rationalization for conservation and age, grade, or sex. The data and chi-square values on which this conclusion was based are presented in Table XIII.

<u>Conclusion</u>. Hypothesis Six was accepted in its entirety. Mode of rationalization for conservation does not seem to be related to the age, grade level, or sex of the students.

TABLE XIII

RELATIONSHIP BETWEEN MODE OF RATIONALIZATION AND AGE, GRADE, AND SEX

Mode of Rationalization	Age		Grade		Sex	
	≤96 mos.	≥96 mos.	Two	Three	Boys	Girls
Operational Identity	19	23	17	25	23	19
Substantive Identity	15	19	16	18	17	17
Reversibility	6	3	5	4	8	1
Compensation	5	5	5	5	7	3
Chi-Square	1.593		0.908		5.186	
Probability of chi-square	.661		.823		.159	

III. RELATIONSHIP OF THE MODE OF RATIONALIZATION TO MATHEMATICS ACHIEVEMENT AND ABILITY

Hypothesis Seven

Hypothesis Seven Restated. There is no significant interaction between the mode of rationalization for conservation and intelligence, vocabulary, listening ability, socio-economic status, age, grade, and sex when mathematical ability and mathematics achievement are the criteria.

Test Procedure. The above hypothesis was tested by a

set of two-way analyses of variance. The test for interaction was carried out simultaneously with the test for main effects in Hypothesis Eight. A more detailed account of the method used is given in the description of the test procedure for Hypothesis Eight.

Results. Tables XX through XXVI in Appendix D contain a summary of the analysis of variance on which the following results were based.

The amount of interaction between the mode of rationalization for conservation and the other independent variables listed in Hypothesis Seven was not significant. Six criteria were used in the above analysis. The criteria were mathematics ability, total mathematics achievement, and the four subtests of the mathematics achievement test. The four subtests were geometry, numeration, addition and subtraction basic facts, and multiplication and division basic facts. Thus there were forty-two tests made for interaction, none of which produced a significant F value.

Conclusion. Hypothesis Seven was accepted. The mode of rationalization did not interact with age, grade, sex, vocabulary, listening ability, socio-economic status, or intelligence to produce any differential effects on mathematics ability or mathematics achievement.

Hypothesis Eight

Hypothesis Eight Restated. There are no significant main effects due to mode of rationalization on the mathematics ability and mathematics achievement tests.

Test Procedure. Hypothesis Eight was tested by the use of a two-way analysis of variance in conjunction with Hypothesis Seven. The computer program used was based on the least-squares technique (Winer, 1962). This seemed to be appropriate since the unequal cell frequencies were a result of the mode of rationalization and were not extraneous to the treatment. The values of the F-ratio were based on the assumption of a fixed model.

Results. Table XIV contains the means and standard deviations of the four rationalization groups on the mathematics ability and the mathematics achievement tests. Tables XX through XXVI in Appendix D contain a summary of the analysis of variance used to arrive at the following results.

Hypothesis Eight was concerned with the main effects due to the mode of rationalization for conservation. There were no significant main effects due to mode of rationalization when mathematics ability, mathematics achievement, and subtests of mathematics achievement were used as criteria. None of the probabilities associated with the observed F-value even

TABLE XIV

MEANS AND STANDARD DEVIATIONS OF THE MATHEMATICS ABILITY AND ACHIEVEMENT TESTS FOR THE FOUR RATIONALIZATION GROUPS

Criterion	Operational Identity		Substantive Identity		Reversibility		Compensation	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Mathematics Ability	43.43	7.79	42.85	9.11	45.67	5.61	43.20	8.05
Mathematics Achievement (Total Score)	24.69	8.32	25.88	7.38	26.22	6.89	25.10	6.90
Geometry	6.57	1.88	6.79	1.51	6.78	1.79	7.30	1.06
Numeration	5.38	2.11	5.76	2.05	6.56	2.01	5.30	2.11
Addition/ Subtraction	8.10	2.36	8.68	1.70	8.44	1.59	8.90	1.20
Multiplication/ Division	4.60	3.47	4.65	3.98	4.44	3.64	3.60	4.06

approached statistical significance.

Conclusion. Hypothesis Eight was accepted. The four groups of subjects formed on the basis of their preferred mode of rationalization for conservation did not differ significantly in their mathematics achievement or in their mathematics ability.

Other Main Effects. Since Hypothesis Eight was tested with a two-way analysis of variance it was also possible to look at the main effects due to other independent variables with mathematics achievement and mathematics ability as the criteria. A summary of the analysis of variance is contained in Tables XX through XXVI in Appendix D.

When the geometry subtest of the mathematics achievement test was the criterion there were significant main effects due to grade at the .03 level of significance. The grade two mean was 6.33 compared to 7.06 for grade three. There were also significant main effects due to intelligence ($p < .001$) as would be expected since there is a positive relationship between achievement and intelligence (see Chapter II). The below average intelligence group had a mean of 5.76 compared to 7.39 for the above average group.

Three groups were formed on the basis of their vocabulary scores by using plus and minus one-half standard

deviation from the vocabulary mean as the cutting points. On the geometry subtest main effects due to vocabulary were significant at less than the .001 level. The means were 5.56, 6.71, and 7.40 for the low, middle, and high vocabulary groups respectively. Listening scores were also divided into three groups by the same method and this variable also resulted in significant main effects ($p < .001$) on the geometry subtest. The means for the low, middle, and high listening ability groups were 5.35, 6.78, and 7.47 respectively.

On the numeration subtest of the mathematics achievement test there were significant main effects due to grade, age, intelligence, vocabulary, and listening ability ($p < .001$ in each instance). The mean on the numeration subtest for grade two subjects was 4.48 compared to 6.38 for the grade three children. Subjects who were ninety-six months of age or younger had a mean score of 4.60 while subjects who were older than ninety-six months averaged 6.33 on the numeration subtest. Below average intelligence subjects had a mean of 4.10 compared to 6.46 for the above average intelligence group. Subjects who had low, medium, and high vocabulary scores had means on the numeration subtest of 3.52, 5.44, and 6.67 respectively. Similarly low, medium, and high listening ability groups averaged 3.96, 5.28, and 6.61 respectively on the numeration aspect of mathematics achievement.

On the addition and subtraction basic facts there were

also significant differences between groups formed on the basis of grade, age, intelligence, and listening ability at beyond the .001 level of significance. There were also main effects due to vocabulary which were significant at the .003 level. Grade three subjects were able to recall significantly more basic facts than grade two subjects. The grade three mean score was 9.15 compared to 7.52 for grade two. The older subjects with a mean of 9.12 out performed their younger counterparts who had a mean of 7.62. Again the above average intelligence group scored significantly better than the below average group. The means were 9.05 and 7.43 respectively. The means on the addition and subtraction basic facts subtest for low, medium, and high vocabulary subjects were 7.36, 8.29, and 9.05 respectively while for low, medium, and high listening ability subjects the means were 6.87, 8.65, and 9.00 respectively.

The main effects on the multiplication and division basic facts subtest were similar to those on the other three subtests. That is, significant main effects were observed for grade, age, intelligence, vocabulary, and listening. All these main effects were significant at beyond the .001 level. Grade two subjects had a mean of only 1.75 compared to 6.66 for the grade three students. Subjects at or below the mean age of 96.5 months averaged only 2.12 on this subtest while their older counterparts

averaged 6.49. Below and above average intelligence subjects scored 2.38 and 5.71 respectively. The means for the low, medium, and high vocabulary groups were 1.08, 4.24, and 6.33 respectively and for the low, medium, and high listening ability subjects the means were 1.61, 4.23, and 5.34 respectively.

When the total mathematics achievement test scores were used as the criterion, the same five independent variables reported above produced main effects which were significant at beyond the .001 level. That is, groups formed on the basis of grade, age, intelligence, vocabulary, and listening ability differed significantly in their achievement in mathematics. The mean for grade two was 20.08 and for grade three it was 29.28. Younger subjects averaged 20.78 on the achievement test compared to 28.96 for older subjects. Below and above average intelligence subjects had means of 19.71 and 28.69 respectively. Low, medium, and high vocabulary groups had means of 17.52, 24.74, and 29.45 respectively while the low, medium, and high listening ability groups averaged 17.78, 25.63, and 28.47 respectively.

When mathematics ability was the criterion there were even more significant main effects. In addition to the five variables which produced significant main effects on mathematics achievement, sex also produced significant main effects on mathematical ability. The main effects

due to grade were significant at the .001 level. The grade two subjects had a mean score of 40.46 while the grade three subjects averaged 45.81 on the mathematics ability test. There were also significant differences ($p = .03$) between the sexes. The boys mean was 44.86 and the girls mean score was 41.12. Differences between the two age groups were significant at the .002 level. Students who were below the mean age of the total sample scored 40.90 while the older students had a mean of 45.59.

Main effects due to intelligence, vocabulary, and listening ability were all significant at beyond the .001 level of significance. Below average intelligence students had a mean mathematical ability score of 38.69 compared to 46.53 for above average intelligence subjects. The means for the three vocabulary groups (low, medium, and high) were 37.80, 43.12, and 46.64 respectively. The low, medium, and high listening ability groups had means of 36.35, 43.50, and 47.21 respectively.

Conservation as a Criterion. Conservation scores were used as a criterion as well as mathematics ability and mathematics achievement. The results of the two-way analysis of variance corroborate the results reported for Hypothesis Five. That is, there were significant main effects due to mode of rationalization for conservation. Tables XX through XXVI in Appendix D also contain a summary

of the analysis of variance with conservation as the criterion.

There were also significant main effects on the conservation test due to intelligence ($p = .001$), vocabulary ($p < .001$), and listening ability ($p = .01$).

Hypothesis Nine

<u>Hypothesis Nine Restated</u>. There are no significant differences among subjects who use different modes of rationalization for conservation in mathematical ability or mathematical achievement when intelligence, vocabulary, listening ability, conservation, socio-economic status, age, grade, and sex are controlled for separately.

<u>Test Procedure</u>. The above hypothesis was tested with a one-way analysis of covariance technique using a multiple linear regression approach. This method compares the predictive ability of two different models (Kelly, 1969). The first model contains information about the mode of rationalization for each subject and the covariate while the second model contains information on only the covariate. If there is a significant difference in the ability of these two models to predict the criterion it is due to differences in the group membership and the differences exist over and above the effects of the covariate since it is contained in both models.

The F-values obtained in the analyses have three degrees of freedom in the numerator and ninety degrees of freedom in the denominator except where socio-economic status was used as a covariate. The degrees of freedom in the latter instance were three and eighty-six in the numerator and denominator respectively.

Results. None of the F-values associated with the analysis of covariance tests reached significance at the .05 level. In other words, the four rationalization groups did not differ significantly on the mathematical ability test, on the mathematics achievement test, or on any of the subtests of the achievement test.

The only analysis which approached significance was a comparison of the four groups on the numeration subtest of the mathematics achievement test with age as the covariate. The F-ratio was 2.149. The probability that an F-value as large or larger than the one observed is about 0.10.

Conservation as a Criterion. It was found in the analysis of Hypothesis Five that subjects with different preferred modes of rationalization for conservation differed significantly in their ability to conserve. Consequently, conservation was used as the criterion in a supplementary analysis of Hypothesis Nine. Table XV contains a summary of the analysis of covariance for each covariate.

The differences among the four rationalization groups in their ability to conserve remains significant when the effects of mathematics achievement, sex, grade, intelligence, age, listening ability, vocabulary, and socio-economic status are statistically controlled one at a time.

TABLE XV

SUMMARY OF ANALYSIS OF COVARIANCE ON CONSERVATION

Covariate	Degrees of Freedom	F	Prob. of F
Mathematics Achievement	3, 90	4.71	.004
Sex	3, 90	4.10	.009
Grade	3, 90	4.26	.007
Intelligence	3, 90	4.06	.009
Age	3, 90	4.39	.006
Listening Ability	3, 90	4.81	.004
Vocabulary	3, 90	3.87	.012
Socio-Economics Status	3, 86	3.97	.011

Hypothesis Ten

Hypothesis Ten Restated. There are no significant differences among subjects who use only one mode, two different modes, three different modes or all four modes of rationalization for conservation in mathematical achievement or mathematical ability.

Test Procedure. Hypothesis Ten was tested by a one-way analysis of variance with a Newman-Keuls comparison of ordered means used to test for differences between pairs of means. Subjects were placed into four groups on the basis of the number of different rationalizations given for conservation over the complete conservation test. Group one (N = 15) used only one mode of rationalization (includes subjects who conserved on only one item), group two (N = 27) used any two of the four modes, group three (N = 45) used any three of the four modes, and group four (N = 15) used all four modes of rationalization.

Results. Table XVI contains the means and standard deviations for the above four groups on the mathematical achievement and mathematical ability tests. In addition, Table XVI contains the F-value and its associated probability for each analysis. The degrees of freedom were three and ninety-eight for the numerator and denominator respectively in each analysis except where socio-economic status was used as the criterion. For the latter analysis the degrees of freedom were three and ninety-three for the numerator and denominator respectively.

(a) Mathematics Achievement. On all four subtests of the achievement test and on the total score, the group who used three different rationalizations had the highest mean score and those who used only one mode had the lowest

mean score. However, differences among the groups were statistically significant for the numeration subtest and for the total test only.

TABLE XVI

COMPARISON OF GROUPS WHICH USED DIFFERENT NUMBERS OF RATIONALIZATIONS ON THE MATHEMATICS ACHIEVEMENT AND MATHEMATICAL ABILITY TESTS

Criterion	Number of Rationalizations					
	One	Any 2	Any 3	All 4	F	Prob. of F
Mathematics Achievement						
Numeration						
Mean	4.40	5.11	6.27	5.53	4.00	.010
S.D.	1.92	2.03	2.04	1.92		
Geometry						
Mean	5.87	6.56	7.09	6.73	2.18	.096
S.D.	1.96	2.01	1.31	1.53		
Add/Subtract						
Mean	7.60	8.00	8.84	8.60	2.08	.107
S.D.	2.59	1.86	1.86	1.50		
Mult./Divide						
Mean	3.33	4.04	5.11	4.27	1.07	.366
S.D.	4.06	3.26	3.71	3.92		
Total Achievement						
Mean	21.33	23.70	27.31	25.13	2.91	.038
S.D.	8.84	7.20	7.40	6.52		
Mathematics Ability						
Mean	39.60	39.78	45.93	44.60	4.96	.003
S.D.	7.36	10.92	6.00	4.87		

When the Newman-Keuls test was applied to the means on the numeration subtest it was found that group one had a significantly lower mean score than group three. None of the other pairs were significantly different.

The Newman-Keuls test resulted in no significant comparisons on the total test means although the means of group one and group three were very close to being significantly different.

Both for the numeration subtest and for the total test the variances were homogeneous.

(b) Mathematical Ability. The differences among the means of the four groups (number of different rationalizations) on the mathematical ability test were significant at the .003 level. The significant differences seemed to be due to the difference between the means of group one and group three since this was the only pair that were significantly different by the Newman-Keuls comparison. Students who used only one mode of rationalization for conservation had the lowest mean mathematical ability score while students who used any three of the four modes had the highest mean score.

However the variance was not homogeneous nor were the data normally distributed. To compensate for this, the analysis was re-run using normalized scores with a mean of 50 and a standard deviation of 10. The end result was almost identical to that reported above. The significance

level was now .002 and the Newman-Keuls test resulted in groups one and three being significantly different.

Other Variables. In addition to the mathematical criteria specified in Hypothesis Ten the groups formed on the basis of the number of rationalizations used for conservation were also compared on conservation, intelligence, listening ability, vocabulary, and socio-economic status. The means and standard deviations for the four groups on these variables are given in Table XVII along with the F-ratio and its probability for each analysis of variance test.

Differences among the four groups on conservation were significant at beyond the .001 level. In this instance it was the group which used all four modes of rationalization that had the highest mean score. As the number of rationalizations used decreased, mean scores on the conservation test decreased so that those who used only one mode of rationalization had the lowest mean conservation score.

An application of the Newman-Keuls test indicated a significant difference between group one and group three, between group one and group four, between group two and group three, and between group two and group four. That is, subjects who used only one mode were significantly lower than all the other groups except for those who used two modes and subjects who used all four modes were significantly

TABLE XVII

COMPARISON OF GROUPS WHICH USED DIFFERENT NUMBERS OF RATIONALIZATIONS ON CONSERVATION, INTELLIGENCE, LISTENING, VOCABULARY, AND SOCIO-ECONOMIC STATUS

Criterion	Number of Rationalizations					
	One	Any 2	Any 3	All 4	F	Prob. of F
Conservation						
Mean	4.67	5.30	6.87	7.40	11.72	<.001
S.D.	2.41	2.02	1.27	1.12		
Intelligence						
Mean	19.67	21.96	25.71	25.87	5.11	.003
S.D.	6.95	5.44	6.38	5.62		
Listening						
Mean	37.20	38.52	40.82	39.87	2.73	.048
S.D.	6.59	4.69	3.71	5.55		
Vocabulary						
Mean	21.80	23.33	26.40	28.13	3.93	.011
S.D.	8.00	6.92	5.58	4.75		
Socio-Economic Status						
Mean	44.14	45.56	49.34	46.61	0.48	.697
S.D.	18.62	15.51	16.43	17.52		

higher in terms of conservation scores than all the other groups except for those who used three different modes of rationalization for conservation.

There were significant differences among the four groups in intelligence, listening ability, and vocabulary scores but they apparently came from the same population in terms of socio-economic status.

On Raven's Coloured Progressive Matrices, the group

of subjects who used only one mode of rationalization had significantly lower scores than the subjects who used three different modes and also lower than subjects who used all four modes. None of the other pairs were significantly different.

On the listening test there were over-all differences that were significant at the .048 level. However, the Newman-Keuls test did not result in any of the pairs of means being significantly different. Group one, as usual, had the lowest score and group three had the highest.

Over-all differences on the vocabulary test were significant at the .01 level of significance. Students who used only one mode of rationalization for conservation had significantly lower vocabulary scores than students who used all four modes and also lower vocabulary scores than subjects who used three different modes.

Conclusion. In terms of mathematical achievement it appears that Hypothesis Ten must be rejected since the groups differed significantly on the total test score. However, when the subtests of the achievement test were considered, there were significant differences on only the numeration subtest.

With respect to the mathematical ability of the subjects, Hypothesis Ten must be rejected. Significant differences at the .003 level existed among the groups.

On both the mathematics ability and mathematics achievement tests it was the group who used only one mode of rationalization for conservation that had the lowest scores and those who used any three of the four modes had the highest scores. This pattern was also consistent when conservation, intelligence, listening, and vocabulary scores were considered.

The findings of Hypothesis Ten have a number of implications which will be discussed in the next chapter.

IV. SUMMARY

Chapter IV contained the results of testing ten hypotheses which were associated with the three major purposes of the present study.

The first purpose of the study was to investigate the frequency with which the different modes of rationalization for conservation were used. It was found that the primary school children used in the present study preferred to use identity-type arguments when they rationalized conservation. Many more subjects responded this way than with reversibility or compensation arguments. This was true for the total sample and for total conservers and for partial conservers separately.

Total conservers and partial conservers also seemed to differ somewhat in the mode of rationalization they used for conservation. Partial conservers tended to use

compensation and substantive identity more than total conservers who in turn used operational identity and reversibility to a greater extent than partial conservers. However, both groups used identity-type arguments most extensively.

The difficulty of the items on the conservation test was related to the mode of rationalization used. The identity mode of rationalization was the most common over all difficulty levels but reversibility was used more on the difficult items than was compensation. Compensation arguments were more frequent than reversibility arguments on the easier items.

The second purpose of the present study was to investigate differences in social and personal characteristics of the students who exhibited different modes of rationalization. There were no significant differences among the four rationalization groups in intelligence, socio-economic status, vocabulary, listening ability, age, grade, or sex. The only characteristic in which the groups differed significantly was the ability to conserve. Students who preferred to rationalize conservation with reversibility arguments had the highest score on the conservation test. Students in the compensation group had the lowest mean on the conservation test.

The major purpose of the present study was to relate the mode of rationalization students used to their mathematical

ability and achievement. The reversibility group had the highest mean score on the mathematics tests but none of the differences among the groups could be considered significant. However, when the subjects were classified by the number of different modes of rationalization they used, significant differences were observed. Generally students who used three different modes of rationalization over the whole conservation test were superior in ability and achievement. On the other hand students who could use only one mode of rationalization were the lowest achievers in mathematics. This observation has implications which will be discussed in the next chapter.

CHAPTER V

SUMMARY, DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

I. SUMMARY OF THE INVESTIGATION

The present study developed from the theory of Jean Piaget. He claims that there are three thought processes which a child may use in deciding that some property of an object is conserved after it has undergone a perceptual but invariant transformation. The three thought processes include identity, reversibility, and compensation. In the present study, sixty grade two and sixty grade three students were given a set of eight conservation tasks and were asked to justify their conservation responses. The above three thought processes were considered as possible modes of rationalization that might be used by the subjects.

Rationalizations of the identity-type were subdivided into two subcategories as a result of a pilot study. Operational identity arguments were those that implied that no change in the property had taken place because the transformation was irrelevant to the property. The operation did not do anything to the property. Substantive identity rationalizations were those that said the objects had not changed. The objects after the transformation were identical to the objects before and so they must possess the same amount

of the property. Reversibility rationalizations were those which implied that the property was the same after the transformation because the transformation could be reversed and everything taken back to the starting point. Compensation arguments implied that the property was conserved because while it might appear to have increased in one dimension it had decreased in another dimension. These two dimensions compensated and the property was conserved.

Subjects were placed into one of these rationalization categories on the basis of their rationalization score. A weight of three was assigned to the first rationalization given to a particular item on the conservation test, a weight of two to the second, and a weight of one to the third and subsequent rationalizations given to a particular item on the conservation test. A subject was placed into the category for which he received the highest score.

Purpose

The study reported here had three purposes. The first purpose was to investigate the frequency with which each of the four modes of rationalization for conservation was used to justify conservation responses.

The second purpose was to investigate differences that might exist in subjects who preferred to rationalize conservation with different types of arguments. The

characteristics chosen for examination were social and personal in nature.

The third purpose of the present study was to compare the mathematics ability and mathematics achievement of grade two and three children who used different modes of rationalization for conservation. This was the major purpose of the study.

Instruments

To gather the necessary data several instruments were employed.

Conservation Test. An eight-item Piaget-type conservation test was developed by the author. This test was used to measure each child's ability to conserve and also the rationalization that he used to justify conservation. The latter was secured by asking the subject, "How can you tell that they are the same?" after each conservation response. The complete test was tape recorded so that the rationalizations could be transcribed later for more detailed study and classification. The items on the conservation test involved four properties, number, length, area, and volume. A KR-20 reliability of 0.771 was obtained for the conservation test.

Mathematics Achievement Test. (See Appendix A.) This instrument was also designed by the researcher. It consisted

of thirty-nine items divided into four subtests, Geometry (nine items), Numeration (ten items), Knowledge of Addition and Subtraction Basic Facts (ten items), and Knowledge of Multiplication and Division Basic Facts (ten items). This was a multiple choice type test (except for the basic facts section) and the test had a KR-20 reliability of 0.893.

Intelligence. Intelligence was measured by Raven's *Coloured Progressive Matrices*, Sets A, Ab, and B.

Mathematics Ability and Listening Ability. These abilities were measured by the *Cooperative Primary Tests* Form 12A developed and published by Educational Testing Service.

Vocabulary. The vocabulary section of the *Wechsler Intelligence Scale for Children* was given to each subject individually immediately after the conservation test.

Socio-Economic Status. The occupation of the father of each subject was taken from the cumulative record cards of the students and quantified by Blishen's revised *Occupational Class Scale*. In the event that the father was deceased or otherwise absent from the home the mother's occupation was used.

Major Findings

A summary of the findings will be discussed in terms of the three purposes of the study. Some of the findings will be discussed more subjectively in the next section of this chapter.

In testing the hypotheses only the four rationalization categories, Operational Identity, Substantive Identity, Reversibility, and Compensation, were used and, for the most part, only subjects who conserved on at least two of the eight items on the conservation test were used.

Frequency of Use of the Different Modes of Rationalization. Four hypotheses were stated under the first purpose of the study.

The first hypothesis stated that there was no significant difference between the observed frequency with which each mode of rationalization was used and a chance frequency distribution.

It was found that the above two distributions were significantly different. In other words, primary students seemed to have a preferred mode of rationalization. The data indicated that their preferences were for the identity modes of rationalization. This was true for the total usable sample of ninety-five subjects and for total conservers (N = 53) and partial conservers (N = 42) separately. Grade level did not affect the above finding.

To test the second hypothesis the frequency distribution of the modes of rationalizations for total conservers and for partial conservers were compared with a chi-square test of independence.

There was a statistically significant relationship between mode of rationalization and the type of conserver the subject was. Both total conservers and partial conservers used the identity modes most often but partial conservers tended to use compensation more than the total conservers did. The total conservers, on the other hand, used reversibility arguments more extensively than partial conservers. Considering only the two identity categories, it was found that a greater proportion of partial conservers used substantive identity than operational identity. The opposite was true of total conservers.

The same relationship held for low socio-economic status students but not for subjects of higher social class standing. The statistical significance of the relationship was also lost when the subjects were subdivided by grade, mathematics achievement, mathematics ability, listening ability, vocabulary, intelligence, sex, and age. On the basis of the total sample, Hypothesis Two was rejected.

Hypothesis Three stated that the mode of rationalization was independent of the degree of conservation. This hypothesis could not be accepted or rejected because when the conservation scores were grouped into the three

categories, 2-3, 4-6, and 7-8, the relationship of conservation to mode of rationalization was significant at the .04 level but when the conservation scores were grouped 2-4, 5-6, and 7-8 there was no longer a statistically significant relationship.

The last hypothesis related to the first purpose of the study resulted in an examination of the relationship between mode of rationalization for conservation and the difficulty level of the items on the conservation test. There were three difficulty levels: easy, moderate, and difficult.

The relationship was significant at beyond the .001 level. Most responses were in the identity modes regardless of the difficulty level although the proportion was greater for difficult items than for easy or moderately difficult ones. Compensation was used more extensively than reversibility on easy and moderate items with the opposite being true for the difficult items.

Items classified by difficulty level conformed very closely to topic. That is, the easy items were the two dealing with conservation of number, the moderately difficult items were the area and volume items, and the most difficult items were those dealing with conservation of length.

Differences in the Characteristics of Students Who Use Different Modes of Rationalization. The second purpose

of the present study was to investigate differences in social and personal characteristics of students in the different categories of rationalization. Two hypotheses, the fifth and sixth, were related to this purpose.

Hypothesis Five stated that there were no significant differences among the four rationalization groups in intelligence, socio-economic status, vocabulary, listening ability, and conservation. This hypothesis was accepted for all the above variables except conservation. That is, there were statistically significant differences among the groups formed on the basis of their mode of rationalization on only the conservation test. Differences on this test were significant at beyond the .01 level. Students who used reversibility rationalizations were the best conservers and those who used compensation were the poorest conservers. A Newman-Keuls comparison of ordered means resulted in not only the reversibility and compensation groups being significantly different but also the reversibility and substantive identity groups. Students who preferred reversibility rationalizations for conservation were not, however, significantly better conservers than the pupils in the operational identity group.

To test the sixth hypothesis, the relationship between mode of rationalization and age, grade, and sex was examined by a chi-square test of independence. None of these relationships was statistically significant.

Therefore Hypothesis Six was accepted in its entirety.

Relationship of the Mode of Rationalization to Mathematics Achievement and Ability. The third purpose of the present study was to relate the different modes of rationalization to achievement in various areas of mathematics and to mathematics ability. Four hypotheses, Hypotheses Seven through Ten, were related to this purpose.

The seventh hypothesis stated that there was no significant interaction between the mode of rationalization used to justify conservation and each of the following variables taken one at a time: intelligence, vocabulary, listening ability, socio-economic status, age, grade, and sex. Each test for interaction was made on six criteria, mathematical ability, total mathematical achievement, geometry, numeration, knowledge of addition and subtraction basic facts, and knowledge of multiplication and division basic facts. The latter four criteria were subtests of the mathematics achievement test.

None of the forty-two tests for interaction produced a significant F-value. Consequently Hypothesis Seven was accepted.

To test Hypothesis Eight the main effects due to mode of rationalization were examined to see if there were significant differences among the four rationalization groups on any of the six mathematical criteria stated above.

No significant differences were observed on any of the criteria. Therefore Hypothesis Eight was accepted.

The ninth hypothesis was similar to the eighth except that intelligence, vocabulary, listening ability, conservation, socio-economic status, age, grade, and sex were added as covariates. It was found that controlling statistically for each of these variables one at a time did not change the results reported for Hypothesis Eight. That is, there were no significant differences among the four rationalization groups in mathematics achievement or mathematics ability.

Since it had been observed earlier that the four groups formed on the basis of their preferred mode of rationalization differed significantly in their ability to conserve it was decided to do a one-way analysis of covariance to adjust for possible differences on other independent variables. This was done as a supplement to Hypothesis Nine. It was found that the four groups still differed significantly in their ability to conserve when the effects due to mathematics achievement, sex, grade, intelligence, age, listening ability, vocabulary, and socio-economic status were controlled one at a time.

The final hypothesis stated that there were no significant differences in mathematical ability or achievement among subjects who used different numbers of rationalizations over the complete conservation test.

Subjects were grouped into the following four categories: (1) those who used only one mode of rationalization for conservation, (2) those who used two different modes, (3) those who used three different modes, and (4) those who used all four modes of rationalization for conservation.

Significant differences were found to exist among the above four groups on the numeration aspect of achievement and on the total achievement score. There were also significant differences on the mathematics ability test. In all instances the group which used three different rationalizations had the highest mean scores and those who used only one rationalization had the lowest mean scores. The same was true when these groups were compared on their conservation, intelligence, vocabulary, and listening scores. They did not differ in their socio-economic status.

II. DISCUSSION OF SOME OF THE FINDINGS

The primary students used in the present study were quite clear in their preference for identity-type rationalizations for conservation. Why should this be? There are perhaps two reasons. The first is that identity rationalizations may be the easiest to voice quickly in a testing situation. An effort was made to overcome this possibility by encouraging the subjects to give other reasons and time was allowed for them to respond the second time. Out of the one hundred and three students who

conserved on at least two items, only forty-nine gave more than one response to at least two items on the conservation test. For those who did respond a second time, the most frequent pattern was for those who gave one type of identity response the first time to give the other type of identity rationalization the second time. If a reversibility or compensation argument was given the first time, the second response was frequently an identity one. Therefore, even with the opportunity to give varied responses, the subjects clearly chose to use the identity mode of rationalization for conservation.

A second reason that may explain why students rationalized conservation mainly with identity arguments is that there was less abstraction involved in giving this type of response compared to giving a reversibility or compensation type of rationalization. The representative identity responses, "It is the same stick", and "You didn't break any of this off", indicate a concentration upon the actual concrete objects involved. To give a reversibility argument the child must be concerned with the operation involved and in compensation he must consider two dimensions at once. The latter two types of reasoning are more abstract and since the objects used in the experiment were before the subject his perception of them likely increased the probability that he would respond in terms of the objects rather than in terms of the applied

operation or in terms of compensatory relationships.

Assuming that this second reason is true, an immediate consequence is that the mode of rationalization a student uses for conservation is developmental. As a subject advances through the concrete operational stage and into the formal operations stage described by Piaget he will probably resort more and more to the more abstract forms of rationalization. The data gathered for the present study were not adequate to check this possibility.

A slight clue to the possibility of differences in the frequency of the use of different modes of rationalization was contained in the study by Wallach, Wall, and Anderson (1967) reported in Chapter II. They found that, in spite of the training they had given in reversibility, reversibility was seldom used as a means of justifying conservation responses. This reluctance or inability of subjects to justify conservation in terms of the reversibility of the applied transformation is confirmed in the present study since only nine of the one hundred and three total or partial conservers used reversibility as their preferred type of justification.

The question was also raised as to how the mode of rationalization might be related to the consistency with which subjects used more than one mode of rationalization per conservation test item. While this was not examined as an hypothesis it was, nevertheless, thought worthy of

investigation. Students were subdivided into two groups, those who gave more than one rationalization on two through four items and those who gave more than one response to more than four items. These groups were subdivided by mode of rationalization and a chi-square test was applied to the cross tabulation. No significant relationship was found to exist between the number of times more than one rationalization was given and the mode of rationalization a subject used to justify conservation. Mode of rationalization was replaced by the other variables used in the study. The only variable to which the frequency of the use of different modes was related to was vocabulary. The probability level of chi-square was .005. Students with vocabulary scores above the mean tended to give two or more rationalizations more frequently than subjects with below average vocabulary scores.

The above finding suggests that students who have a good vocabulary from which to draw are better able to justify their conservation responses with a variety of reasons.

It was interesting to observe that only about one-half of the subjects used in the present study were able to conserve length while 69 per cent conserved on the area and volume items. Conservation of length should precede conservation of area or volume and consequently a

greater proportion of students should conserve on the length items than on the area and volume items. However, it may not be valid to talk about conservation of length, area, or volume in the present study since there were only two items on the conservation test which dealt with each of these topics.

On the other hand, it may be that misleading perceptual cues were more pronounced in the items dealing with length than in tasks presented in other studies in that the rods used were long and slender. They were about one foot long and only about one-eighth of an inch in diameter. If during the transformation students focus on the leading end of the moving rod then the greater the probability that the compensating dimension will be ignored. This idea is presented as an hypothesis only.

Piaget has placed considerable emphasis upon the importance of reversibility in the acquisition of conservation. It was observed in the present study that students who preferred to rationalize conservation with reversibility-type arguments had higher conservation scores than students who rationalized conservation with other types of arguments. This finding adds some weight to the emphasis on the role of reversibility in conservation made by Piaget and also by Wallach and Sprott (1964) whose research was briefly outlined in Chapter II.

The finding that the four rationalization groups did not differ in the other social and personal characteristics investigated does not imply that differences do not exist. Different modes of rationalization for conservation may reflect to some extent differences in reasoning styles or thought processes. If this is true then it seems reasonable to suspect that there should be some differences in the characteristics of students who exhibit different modes of rationalization. Perhaps the measuring instruments used in the present study were not adequate to pick up these differences or perhaps other variables should be considered. Since the investigation of the relationship of mode of rationalization for conservation to social and personal variables was not the major purpose of the present study only a few variables were included.

The major purpose of the study was to examine the relationship between the different modes of rationalization for conservation and the achievement of students in various areas of mathematics. It was anticipated that a hierarchy of modes might be constructed for each of the four areas of achievement studied. For example, in geometry students who think in terms of compensation might be the highest achievers followed by, say, the reversibility group whereas in the understanding of numeration a different order may be observed. If hierarchies of this type could have been established, then some implications for curriculum design

and teaching strategies could have been outlined. However, no differences among the four groups existed on any of the mathematical criteria.

The differences between the four rationalization groups on the mathematics tests were not statistically significant. However, there was a tendency for the reversibility group to have the highest means. They obtained the highest mean score on both the mathematics ability test and the total mathematics achievement test but when scores on the subtests of the mathematics achievement test were examined no pattern was evident.

One of the most interesting findings in the present study was related to the effects of the use of multi-modes. It was observed that significant differences existed in mathematics achievement and ability among subjects who used different numbers of rationalizations for conservation. Students who used three different modes had the highest mean scores followed by those who used all four modes of rationalization for conservation.

The above results seem to indicate the importance for success in mathematics of the ability to analyze a problem from different points of view. Students with a limited number of ways of thinking about a problem have less of a chance for success than students who are able to bring a variety of approaches to the problem situation. This has an important implication for teaching which will

be discussed in the following section of the present chapter.

III. SOME IMPLICATIONS OF THE FINDINGS

For Teachers

Piaget does not have much to say about the implications of his theory for education. Other writers have attempted to draw these out. For example, Sullivan (1967) lists four major uses of Piaget's theory in education. He says that it could be used as a method of assessing intellectual capacity, as an aid in the structure and sequencing of the subject matter within the curriculum, as an aid in assessing learning outcomes in a curriculum, and as suggestive of the types of learning atmospheres most conducive to learning in young children.

Piaget's studies and most replication studies have shown that the type of tasks used in a test of conservation are useful for assessing readiness for concepts. If a child does not see the invariance of a given number of objects in different spatial arrangements he is not likely to gain maximum benefit from a study of ordinal or cardinal number, or the basic facts.

The question, "How can you tell?", has often been used in studies as a check on the validity of a child's conservation response. If a subject gave a rational answer his conservation response was considered meaningful,

if an irrational answer was given it was assumed the child was a nonconserver and responded, "Yes", only because he thought the experimenter wanted him to. It was hoped that the type of answer to the question, "How can you tell?", would be more useful than that and provide some implications for classroom teaching in the sense that the type of rationalization could be used to predict achievement in mathematics. However, this was not the way the results turned out.

One implication does arise for teachers concerned with teaching conservation if we assume that teaching explicitly for conservation is desirable. Since most students rationalized conservation with identity-type arguments it would seem reasonable that in providing training for the purpose of inducing conservation, experiences should be provided which emphasize the identity aspects. For example, in teaching conservation of continuous quantity questions such as the following could be asked:

"Is it the same water?"

"Did I add any water to this jar?"

"Where did I put the water that I took away?"

(Of course no water was added or taken away.)

The above approach to inducing conservation has some resemblance to Smedslund's "cognitive conflict" approach in which a child was directed into a situation

where his perception conflicts with his understanding that no addition or subtraction of the property had taken place. Perhaps some modification of this approach may be the most successful for speeding up the acquisition of conservation.

The largest number of responses to all items on the conservation test except for item seven (the water task) were in terms of identity. Fifty-two per cent of the responses to the water item were in terms of compensatory relationships. This has implications for teaching volume or capacity. For this particular topic it would seem to be important that experiences be given children which would accent compensatory dimensions. Children should have many opportunities to pour liquid from one container into identical and also into dissimilar containers. Questions need to be posed that would help the child see that the level of a specific amount of liquid in a container is affected by the cross section of the container, that is, changes in level or height is compensated for by changes in cross sectional area.

Students who used three or four different modes of rationalization for conservation were clearly more successful in mathematics than those who used only one or two modes of rationalization. It was mentioned in the discussion section of this chapter that, on the basis of the above finding, the ability to approach a problem from

different points of view was desirable. The implication for teaching is that we should not teach particular rules for particular types of problems but rather we should teach a more general problem-solving approach so that in any particular problem situation a student is not limited by the one rule he knows but rather is able to draw upon all his relevant background knowledge.

For the Curriculum

The two findings that young children think strongly in terms of identity relationships and the tendency for those who think in terms of reversibility to have the highest scores on achievement tests together have implications for sequencing the subject matter.

Consider for example the teaching of the basic facts. Since primary students think in terms of identity it would seem logical to teach all the basic facts for a given number together. Furthermore, since there is the tendency for high achievers to think in terms of reversibility as well as identity the addition and subtraction basic facts should be taught together since one is the inverse of the other. In combining these two ideas the suggestion can be made that all the addition and subtraction basic facts for N should be taught together. The same applies to multiplication and division.

To illustrate, suppose the basic facts for seven were being taught. A child could be given a set of seven counters. He could separate this set into two piles, one with one in it and the other with six. This would illustrate both the fact that $7 - 1 = 6$ and $1 + 6 = 7$. Other arrangements of the counters could illustrate the other basic facts for seven. The identity aspect is apparent since $1 + 6$, $2 + 5$, and so on, have the identical sum of seven and the reversibility aspect is incorporated by introducing subtraction and addition as inverse operations.

Multiplication and division basic facts could be treated in much the same way by working with equal groups of objects.

It was suggested earlier that the identity rationalizations were given most frequently because they related more directly to the objects in front of the students. If this is true then it appears that students could be assisted in their learning of mathematics, or at least in the verbalization of mathematical concepts, by the use of concrete aids. Of course, good teachers use many different aids in their classroom but curriculum designers and textbook publishers have moved very slowly in this direction. The apparent tendency for subjects to rationalize in terms of the concrete situation in front of them may be one more indication that the laboratory approach to learning mathematics could be a fruitful approach.

IV. RECOMMENDATIONS FOR FURTHER RESEARCH

No attempt has been made before to relate style of rationalization for conservation to mathematics achievement or mathematics ability. Therefore this study was intended to be descriptive in many respects.

Since there were clearly no differences among the rationalization groups in their achievement in mathematics, one recommendation for further research is the replication of the present study using different definitions for the categories of rationalization or perhaps a different approach to the coding of responses. Another improvement in a replication study would be to use more items on the conservation test for each topic. It is rather difficult to say that a child who conserves on only two number tasks is a conserver of number. Another recommendation for design improvement is to control the testing environment. In the present study the individual testing situation was nearly the same for all subjects since the testing was done individually and the procedure had been standardized as much as possible. However, due to the kinds of facilities available in the different schools, the group testing situation was not uniform. Perhaps the subjects should be brought to a common testing environment.

A second major recommendation for further research is that a longitudinal study be undertaken in which change

in the mode of rationalization with advancing age is examined. This would find out not only if there is a change but also which mode of rationalization is representative of a more advanced cognitive structure.

A third recommendation for further research is to test the effectiveness of training subjects to recognize identity relationships in inducing conservation. The effectiveness of this method could be compared to the effectiveness of training subjects in reversibility, in compensation, and perhaps in all three modes of thinking about conservation.

Another possible avenue of research might be in the area of social and personal characteristics which distinguish between students who rationalize conservation differently.

When consideration is given to factors which might have affected the results of the present study three factors which require further research seem to stand out. The first is the role of kindergarten experiences in determining the mode of rationalization a child will use for conservation. Students who have been through kindergarten have probably had more learning experiences. How might this have affected how they rationalized conservation and its relationship to mathematics achievement?

The second factor which needs additional research is the effect of previous mathematics experiences. Do the

different strategies used in different textbooks influence the types of rationalizations students give for conservation? This was somewhat controlled in the present study since all the classes were using the same texts. However, different teachers also use different approaches with a given textbook which could influence the way children think about a task such as a conservation problem. In other words some experimental research should be done to see how different teaching strategies affect the mode of rationalization used to justify conservation.

The last factor which needs further investigation is the effect of language upon the mode of rationalization and upon mathematics achievement. The correlation between vocabulary and mathematics achievement was 0.66 in the present study. Furthermore, it was pointed out on page 146 that a student's vocabulary score was significantly related to his ability to give more than one rationalization per conservation test item. Perhaps different ways of rationalizing conservation may be related to the type of language structure a student uses. Research into the above relationships is envisioned as a cooperative effort between mathematics educators and language educators.

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A P P E N D I X   A

MATHEMATICS ACHIEVEMENT TEST

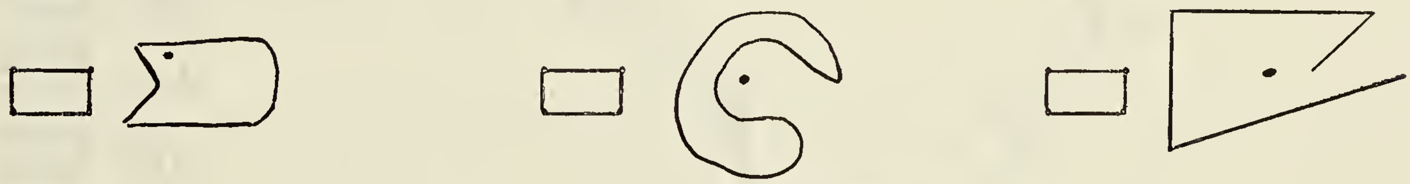
ARITHMETIC TEST

Name ______________

Grade ______________

1. Put an 'X' in the box beside the picture of an open curve.

2. Put an 'X' in the box beside the picture that shows a dot inside a curve.



3. Put an 'X' beside the numeral that has the smallest value.

☐ 111 ☐ 100 ☐ 101 ☐ 110

4. Put an 'X' in the box beside the true statement.

☐ 1 foot is longer than 24 inches.

☐ 1 yard is longer than 3 feet.

☐ 10 inches is longer than 12 inches.

☐ 17 inches is longer than 16 inches.

5. Put an 'X' beside the numeral that means three tens and six ones.

☐ 63 ☐ 36 ☐ 31

6. 47 is between ________ and 76

☐ 36 ☐ 49 ☐ 67

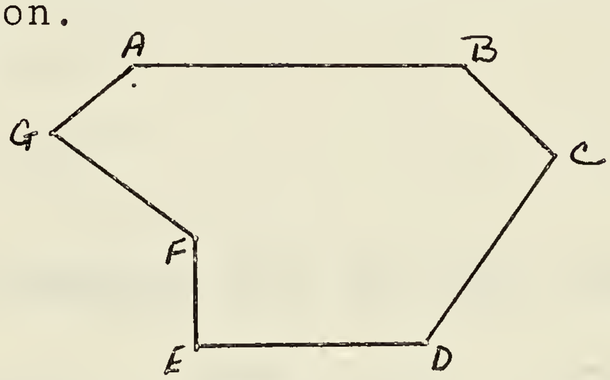
7. Which sign makes this statement true? 4 + 6 = 2 ___ 5

☐ + ☐ - ☐ x

8. Which sign makes this statement true? 82 ____ 74

☐ < ☐ > ☐ =

9. Put an 'X' in the box beside the segment that is a name of a side of the polygon.



☐ DE

☐ FA

☐ AD

☐ CE

10. Put an 'X' beside the numeral which has a three in the hundreds' place.

☐ 2307 ☐ 3725 ☐ 1032 ☐ 2413

11. Put an 'X' beside the sign that makes this statement true. 17 ____ 4 = 13

☐ ÷ ☐ + ☐ x ☐ -

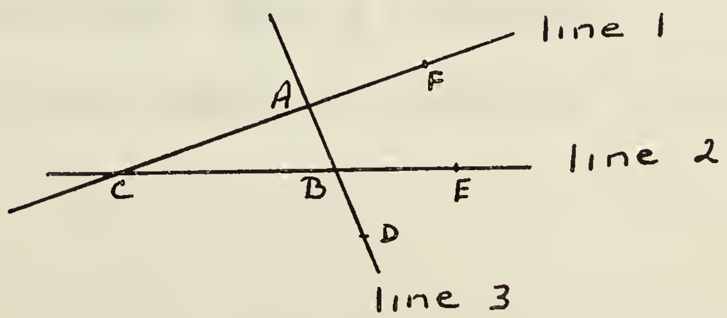
12. Put an 'X' beside the numeral that tells how many sides a triangle has.

☐ 2 ☐ 3 ☐ 4 ☐ 5

13. Put an 'X' in the box beside the letters that name a segment of line 3.

☐ AB ☐ CB

☐ BE ☐ AF



14. Put an 'X' on the glass that has the most water in it.

15. Another way to write 4 x 17 is:

☐ 4 x 10 x 7

☐ 4 x 7 x 10

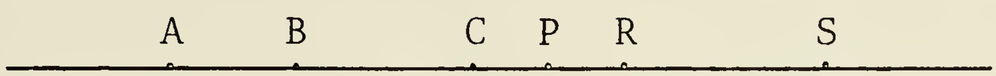
☐ (4 x 10) + (4 x 7)

☐ (4 + 10) x (4 + 7)

16. Put an 'X' beside the numeral that has the largest value.

☐ 2,135 ☐ 1,235 ☐ 5,321 ☐ 5,123

17. Mark the box that names two points that P is between.



☐ A and C ☐ B and C

☐ A and R ☐ R and S

18. Which sign makes this statement true? 20 x 5 ____ 25 x 5

☐ < ☐ > ☐ =

19. Put an 'X' beside the true statement.

☐ A square has more sides than a rectangle.

☐ A square has fewer sides than a triangle.

☐ A square has the same number of sides as a rectangle.

Put the answer to the following questions in the blank after each question.

20. 5 + 3 = ________	30. 7 - 4 = ________
21. 6 x 2 = ________	31. 8 ÷ 2 = ________
22. 13 - 7 = ________	32. 5 + 7 = ________
23. 12 ÷ 3 = ________	33. 5 x 3 = ________
24. 9 - 5 = ________	34. 16 ÷ 4 = ________
25. 5 + 5 = ________	35. 9 - 0 = ________
26. 14 ÷ 2 = ________	36. 7 + 4 = ________
27. 4 x 5 = ________	37. 18 ÷ 9 = ________
28. 3 x 6 = ________	38. 1 x 8 = ________
29. 8 + 8 = ________	39. 13 - 13 = ________

A P P E N D I X B

INTERCORRELATION MATRIX
FOR
MATHEMATICS VARIABLES

TABLE XVIII

INTERCORRELATIONS* AMONG MATHEMATICS VARIABLES

Variable	1	2	3	4	5	6
1. Total Achievement	1.000					
2. Geometry	.711	1.000				
3. Numeration	.824	.529	1.000			
4. Add/Subtract	.776	.485	.539	1.000		
5. Multiply/ Divide	.884	.459	.623	.557	1.000	
6. Mathematics Ability	.727	.505	.652	.637	.578	1.000

* All correlations are significantly different from o at beyond the .001 level.

A P P E N D I X C

INTERCORRELATION MATRIX

FOR

ALL MAJOR VARIABLES

TABLE XIX

INTERCORRELATIONS AMONG MAJOR VARIABLES

Variable	1	2	3	4	5	6	7	8
1. Conservation	1.000							
2. Mathematics Achievement	.392	1.000						
3. Mathematics Ability	.376	.727	1.000					
4. Intelligence	.349	.629	.637	1.000				
5. Socio-economic Status	.204*	.189*	.248*	.221*	1.000			
6. Age	.209*	.540	.392	.323	-.154**	1.000		
7. Listening Ability	.388	.622	.637	.556	.238*	.353	1.000	
8. Vocabulary	.429	.659	.564	.583	.236*	.333	.633	1.000

All correlations are significant beyond the .001 level with the following exceptions:

* -Significant beyond the .05 level.
** -Not significant.

A P P E N D I X   D

SUMMARIES OF TWO-WAY ANALYSES OF VARIANCE

TABLE XX

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR GRADE VS. MODE OF RATIONALIZATION

Criterion	Error		Grade				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	93	2.84	1	13.85	4.88	.03	3	1.62	.57	.56	3	.94	.33	.71
Numeration	93	3.67	1	95.81	26.10	<.001	3	5.90	1.61	.17	3	.44	.12	.84
Add/Subtract	93	3.11	1	71.03	22.84	<.001	3	4.95	1.59	.17	3	7.58	2.44	.06
Multiply/ Divide	93	7.90	1	610.28	77.28	<.001	3	2.95	.37	.68	3	3.88	.49	.61
Total Achievement	93	39.63	1	2191.62	55.30	<.001	3	31.72	.80	.44	3	3.81	.10	.85
Mathematics Ability	93	59.72	1	743.13	12.44	.001	3	26.85	.45	.64	3	40.42	.68	.50
Conser- vation	93	4.48	1	6.10	1.36	.25	3	17.59	3.93	.01	3	.70	.16	.82

TABLE XXI

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR SEX VS. MODE OF RATIONALIZATION

Criterion	Error		Sex				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	93	2.97	1	1.29	.43	.51	3	1.22	.41	.66	3	.85	.29	.74
Numeration	93	4.59	1	7.69	1.68	.20	3	3.13	.68	.50	3	1.48	.32	.72
Add/Subtract	93	3.94	1	3.94	1.00	.32	3	3.50	.89	.40	3	4.19	1.06	.33
Multiply/ Divide	93	14.21	1	28.83	2.03	.16	3	3.11	.22	.78	3	1.88	.13	.83
Total Achievement	93	61.53	1	131.11	2.13	.15	3	8.22	.13	.83	3	11.93	.19	.80
Mathematics Ability	93	64.62	1	304.75	4.72	.03	3	5.83	.09	.86	3	34.60	.54	.58
Conser- vation	93	4.35	1	16.48	3.79	.06	3	15.42	3.55	.02	3	1.12	.26	.76

TABLE XXII

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR AGE VS. MODE OF RATIONALIZATION

Criterion	Error		Age				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	93	2.92	1	7.73	2.65	.11	3	1.51	.52	.60	3	.30	.10	.85
Numeration	93	3.80	1	83.19	21.90	$<$.001	3	6.66	1.75	.14	3	.70	.18	.80
Add/Subtract	93	3.24	1	57.91	17.86	$<$.001	3	4.01	1.24	.27	3	7.80	2.41	.06
Multiply/ Divide	93	9.13	1	488.12	53.49	$<$.001	3	3.94	.43	.65	3	6.50	.71	.49
Total Achievement	93	44.09	1	1746.96	39.63	$<$.001	3	30.86	.70	.49	3	13.99	.32	.72
Mathematics Ability	93	61.41	1	601.42	9.79	.002	3	35.22	.57	.56	3	35.44	.58	.56
Conser- vation	93	4.49	1	4.35	.97	.33	3	18.41	4.10	.008	3	.87	.19	.80

TABLE XXIII

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR INTELLIGENCE VS. MODE OF RATIONALIZATION

Criterion	Error		Intelligence				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	93	2.23	1	71.02	31.80	$<.001$	3	3.39	1.52	.19	3	.60	.27	.75
Numeration	93	3.32	1	126.72	38.19	$<.001$	3	.81	.24	.77	3	1.09	.33	.71
Add/Subtract	93	3.25	1	67.96	20.89	$<.001$	3	4.72	1.45	.21	3	4.14	1.27	.26
Multiply/ Divide	93	11.53	1	275.55	23.90	$<.001$	3	3.08	.27	.75	3	2.80	.24	.77
Total Achievement	93	42.15	1	1962.64	46.56	$<.001$	3	18.79	.45	.64	3	2.10	.05	.87
Mathematics Ability	93	51.80	1	1481.25	28.59	$<.001$	3	11.46	.22	.78	3	39.92	.77	.46
Conser- vation	93	3.86	1	47.00	12.18	.001	3	14.70	3.81	.01	3	6.20	1.61	.17

TABLE XXIV

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR SOCIO-ECONOMIC STATUS VS. MODE OF RATIONALIZATION

Criterion	Error		SES				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	89	2.71	1	10.16	3.75	.06	3	1.62	.60	.55	3	2.10	.78	.45
Numeration	89	4.46	1	8.18	1.84	.18	3	3.86	.87	.41	3	5.90	1.32	.24
Add/Subtract	89	3.78	1	.86	.23	.63	3	1.88	.50	.61	3	1.52	.40	.67
Multiply/ Divide	89	14.56	1	4.66	.32	.57	3	1.34	.09	.86	3	2.76	.19	.80
Total Achievement	89	61.57	1	50.50	.82	.37	3	6.82	.11	.85	3	13.92	.23	.78
Mathematics Ability	89	64.60	1	214.81	3.33	.07	3	18.21	.28	.74	3	8.34	.13	.84
Conser- vation	89	4.56	1	4.38	.96	.33	3	16.26	3.57	.02	3	2.39	.53	.59

TABLE XXV

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR VOCABULARY VS. MODE OF RATIONALIZATION

Criterion	Error		Vocabulary				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	89	2.26	2	26.32	11.62	<.001	3	1.16	.51	.60	6	4.39	1.94	.08
Numeration	89	3.23	2	73.95	22.92	<.001	3	1.77	.55	.58	6	.58	.18	.98
Add/Subtract	89	3.66	2	22.21	6.06	.003	3	3.40	.93	.38	6	2.06	.56	.76
Multiply/ Divide	89	9.73	2	219.95	22.62	<.001	3	4.29	.44	.64	6	8.47	.87	.52
Total Achievement	89	39.43	2	1105.59	28.04	<.001	3	4.53	.12	.84	6	28.11	.71	.64
Mathematics Ability	89	56.63	2	592.01	10.45	<.001	3	5.55	.10	.85	6	32.38	.57	.75
Conser- vation	89	3.90	2	34.53	8.87	<.001	3	18.43	4.73	.004	6	1.47	.38	.89

TABLE XXVI

SUMMARY OF ANALYSIS OF VARIANCE ON SEVEN CRITERIA FOR LISTENING VS. MODE OF RATIONALIZATION

Criterion	Error		Listening				Mode				Interaction			
	DF	MS	DF	MS	F	P	DF	MS	F	P	DF	MS	F	P
Geometry	89	2.31	2	34.35	14.86	<.001	3	2.62	1.13	.30	6	1.02	.44	.85
Numeration	89	3.50	2	52.97	15.15	<.001	3	5.13	1.47	.20	6	3.58	1.02	.42
Add/Subtract	89	3.17	2	38.76	12.23	<.001	3	6.10	1.92	.12	6	3.90	1.23	.30
Multiply/ Divide	89	11.24	2	114.58	10.19	<.001	3	3.88	0.35	.70	6	21.07	1.87	.09
Total Achievement	89	42.59	2	873.17	20.50	<.001	3	36.74	.86	.41	6	58.73	1.38	.23
Mathematics Ability	89	48.66	2	866.64	17.81	<.001	3	32.76	.67	.51	6	59.11	1.22	.31
Conser- vation	89	4.07	2	19.89	4.88	.01	3	16.51	4.05	.008	6	3.73	.92	.49

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